ISCTE — INSTITUTO UNIVERSITÁRIO DE LISBOA

Master in Economics

Macroeconomics

Final Test / **Exam**

5 December 2016 **Duration: 2 hours**

Group A – 50 points (25 points each question)

Question 1 (Matlab). Compulsory for those taking the Exam
Using your knowledge about the Matlab package, write down a simple code in order to:

1. To represent a matrix A of type (3×3) :

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 4 & 6 \\ 2 & 0 & 9 \end{bmatrix}.$$

2. To calculate the determinant of A, the inverse of A, the eigenvalues of A, the list of all elements of column 1 of A, and the list of all elements of line 2 of A.

3. To obtain a solution to $\mathbf{E}_t \mathbf{z}_{t+1}$, given the following expression

$$\mathbf{A} \cdot \mathbf{E}_t \mathbf{z}_{t+1} = \mathbf{B} \cdot \mathbf{z}_t + \mathbf{C} \cdot \mathbf{v}_t$$

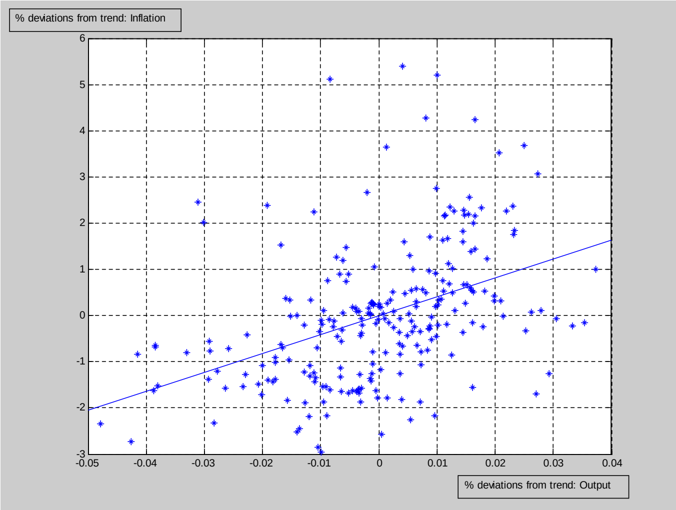
   where $\mathbf{z}_{t+1}, \mathbf{z}_t, \mathbf{v}_t, \mathbf{C}$ are vectors, while $\mathbf{A}$ and $\mathbf{B}$ are matrices.

4. To show the dynamics of the following stochastic processes (x_t, y_t):

$$\begin{aligned} x_{t+1} &= 2 + 0.5x_t + \varepsilon_t \\ y_{t+1} &= 100 + x_{t+1} + \varepsilon_t \end{aligned}$$

where ε_t is a IID random variable, with mean equal to zero and variance equal to 1. In Matlab this random variable is written as: `randn(1)`. Simulate the dynamics of this process for $t = [1, 80]$ in a `yyplot`.

Question 2 (HP filter). Consider the following figure, which represents the percentage deviations from trend of real GDP and the inflation rate, for the US economy between 1947 and 2014 (the straight line represents the least squares regression line) Those % deviations were calculated by using the Hodrick–Prescott filter (HP filter).



The HP filter is obtained through the formula

$$\min_{\tau_t} \sum_{t=1}^{T} \{(y_t - \tau_t)^2 + \lambda[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2\}$$

where y_t is the original series, τ_t is the smooth trend and $(y_t - \tau_t)$ is the HP filtered series and λ is a well known parameter.

1. What happens to the filter in two extreme cases $\lambda = 0$, $\lambda \to \infty$?
2. What are the major advantages regarding the linear trend, and the linear trend with breaks?
3. What are the main limitations of this filter. Explain.
4. What is the relevance of the relationship above for macroeconomic policy?

Question 3 (IRF). Describe what we call in macroeconomics by an Impulse Response Function? Why are they of large usefulness in macroeconomics?

Group B (50 points)

Real Business Cycles. Assume a small scale Real Business Cycle model, the dynamics of which can be described by four fundamental equations, as follows:

$$\begin{aligned} Y_t/N_t &= [\xi/(1-\alpha)]\, C_t \\ Y_t &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha} \quad , \quad \alpha = 0.5 \\ \ln A_t &= (1-\rho)\ln A^* + \rho \ln A_{t-1} + \varepsilon_t \quad , \quad |\rho| < 1 \\ Y_t &= C_t + I_t \end{aligned}$$

Capital letters represent variables measured in actual values, and the symbols closely follows materials covered in classes. It is also known that in the steady state the following information holds: the rates of growth of K_t and I_t are both equal to 4%, and the rate of growth of N_t is 1% .

1. Rewrite this small version of the model with variables expressed in terms of growth rates**.**
2. Calculate, relatively to the "steady state", the values for the growth rates of the endogenous variables (remember, assuming that $|\rho| < 1$).
3. Assume now that the rate of growth of K is given by the following equation

   $$k_t = \lambda E_t k_{t+1} + x_t \quad , 0 < |\lambda| < 1$$

   where x_t represents the the output gap. Determine the equilibrium level of k_t.
4. Now consider also that

   $$x_t = 1\% + 0.5 x_{t-1} + \varepsilon_t$$

   with ε_t as white noise. Recalculate all growth rates in the steady state, given this new information.

Group C (50 points)

Rules versus Discretion. Assume that the Central Bank's loss function is given by the following function:

$$L = \beta\,(u - u^*) + \gamma\,(\pi - \pi^*)^2$$

u is the unemployment rate, γ and β are parameters, and π is the inflation rate. An asterisk is used to represent the central bank's desired values for each variable.
We know that the behavior of the supply side of the economy can be described by the following Phillips Curve with a supply a shock given by e_t:

$$u = u^n - a(\pi - \pi^e) + e_t$$

where u^n is the natural level of unemployment, π^e is the level of expected inflation, and $a = 15$. Finally assume that private agents have rational expectations

$$\pi^e = \pi.$$

1. Explain the logic behind the Loss function above, as far as the targets of the central bank are concerned.
2. Assuming that $u^* = 5\%, \pi^* = 0$, determine the level of optimal inflation in the case of discretionary behavior by the central bank.
3. Determine the same as in the previous question, but now having the central bank displaying commitment to maintain inflation at its target level
4. .
5. Explain why the result in (3) is better than the result in (2).
6. Does the result in (4) hold, if private agents formulate expectations according to the hypothesis of adaptive expectations. Explain.

Group D (50 points)

IS function. Consider the following Euler equation in the New Keynesian Model

$$u'(C_0) = \beta \cdot \left[u'(C_1) \frac{(1 + r_0)}{(1 + \pi_1)} \right]$$

where r_0 is the short term nominal interest rate, π_1 is the inflation rate between periods 0 and 1, and β is the subjective intertemporal discount factor. C stands for consumption.

1. Using the following CRRA utility function

$$U(c_t) = \frac{C_t^{1-\sigma}}{1-\sigma}, \ \sigma > 0$$

   and considering uncertainty in the model, determine the IS function in this model expressed in terms of percentage deviations from the long term trend.
2. Explain what happens in the current period if private agents expect the starting of an upturn of economic activity next year.
3. Explain what happens if there is an increase in expectations about inflation next year.
4. Assume that next year's inflation expectations go up by 1 percentage points. Taking into account that the Central Bank wants to control inflation, by how much should this bank change its short term interest rate, and when? Explain the logic of your answer.

Group E (50 points) Compulsory

NKM and determinacy. Assume the New Keynesian Model represented by the usual structure: an Aggregate Supply function (AS), the demand side function (IS), and a simple rule for interest rate policy. The symbols are as follows: π_t is the inflation rate, x_t is the output gap, i_t is the nominal short term interest rate to be fixed by the central bank.

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \lambda x_t \\ x_t &= E_t x_{t+1} - a\left(i_t - E_t \pi_{t+1}\right) \\ i_t &= 2\% + v E_t \pi_{t+1} \end{aligned}$$

where (β, λ, a, v) are parameters.

1. Rewrite the model in matricial form, in order to study its stability.That is write the model according to this rule

   $$\mathbf{A} \cdot \mathbf{E}_t \mathbf{z}_{t+1} = \mathbf{B} \cdot \mathbf{z}_t + \mathbf{C} \cdot \mathbf{v}_t$$

   where $\mathbf{z}_{t+1}, \mathbf{z}_t, \mathbf{v}_t, \mathbf{C}$ are vectors, while $\mathbf{A}$ and $\mathbf{B}$ are matrices.

2. For the following set of parameter values $\beta = 0.9$, $\lambda = 0.5$, $a = 2.7$, study the stability of this model for the scenario:

   $$\text{Scenario A}: \ v = 0.2$$

   knowing that $A^{-1} \times B = \begin{bmatrix} \frac{1}{\beta} & -\frac{1}{\beta}\lambda \\ -\frac{1}{\beta}(a - av) & \frac{1}{\beta}\lambda(a - av) + 1 \end{bmatrix}.$

3. What would you conclude about the main message of the New Keynesian Model as far as the fight against inflation is concerned? Propose alternatives (no numerical support for these alternatives is required).

The end of the test

$$A = \begin{bmatrix} \beta & 0 \\ (a - av) & 1 \end{bmatrix}, inverse: \begin{bmatrix} \frac{1}{\beta} & 0 \\ -\frac{1}{\beta}(a - av) & 1 \end{bmatrix} = A^{-1}, \text{ inverse: } A$$

$$B = \begin{bmatrix} 1 & -\lambda \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} \frac{1}{\beta} & 0 \\ -\frac{1}{\beta}(a - av) & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\lambda \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{1}{\beta}\lambda \\ -\frac{1}{\beta}(a - av) & \frac{1}{\beta}\lambda(a - av) + 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{0.9} & -\frac{1}{0.9}0.5 \\ -\frac{1}{0.9}(2.7 - 2.7 \times 0.2) & \frac{1}{0.9}0.5(2.7 - 2.7 \times 0.2) + 1 \end{bmatrix}, \text{ eigenvalues: } 2.125\,5, -0.203\,29$$

The end of the test