The New Keynesian Model: Main Functions

Vivaldo M. Mendes

ISCTE — Lisbon University Institute

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I – What's is the New Keynesian Model?

The Old and the New Keynesian Models

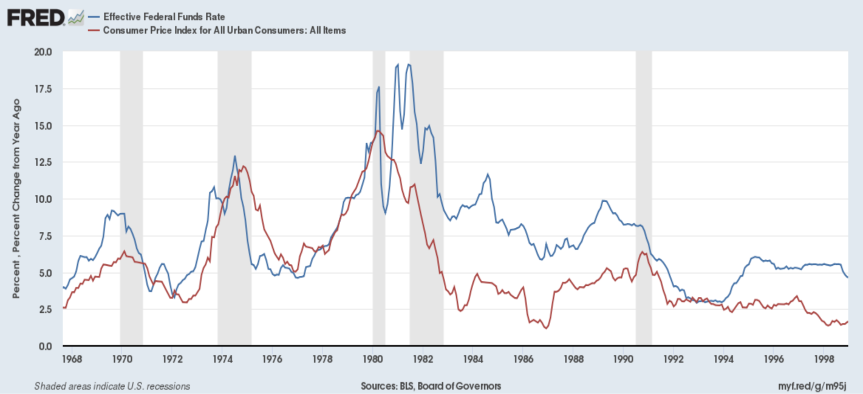
1. The new model is the old Keynesian model
2. **With the usual** nominal and/or real rigidities in price setting
   - Prices are sticky (the baseline version)
   - Nominal wages are rigid downwards
   - Staggered contracts
   - Capacity utilization constraints
3. **Without the problems** that pushed the model to serious trouble in the early 70s
   1. "forward looking/rational expectations" instead of "adaptive expectations"
   2. built upon sound microeconomic foundations
   3. no **permanent** trade-off between inflation and unemployment
   4. no room for stagflation
4. **A new message**: *rules* instead of *discretion* by Central Banks in the management of monetary policy

Four basic predictions

1. **The same usual functions** (IS, LM, Aggregate Supply) ...
2. **Quantitative simulations:** crucial element like in the Real Business Cycle literature
3. **Contrary to RBC**, there is a key role to monetary policy and a minor role for fiscal policy
4. **Four basic predictions** (the old model up-side-down):
    1. the instrument of monetary policy ought to be the short term interest rate,
    2. policy should be focused on the control of inflation,
    3. inflation can be reduced by aggressively increasing short term interest rates (**see next figure**).
    4. the central bank should conduct monetary policy adopting a strategy of commitment in a forward-looking environment, instead of discretion.

Active interest rate policy by the Fed

The FED now reacts much more aggressively to inflation than in the "old times"



A picture of the New Keynesian model

1. On the demand side:
   1. the **IS function** is forward looking
   $$x_t = -\varphi(r_t - E_t\pi_{t+1}) + E_t x_{t+1} + \mu_t$$
   2. the **LM function**, where the central bank now controls the interest rate, not the money supply:
   $$r_t = L(?, ?)$$
2. In the supply side: the **Aggregate Supply function** is also forward looking:
$$\pi_t = \beta \cdot E_t\pi_{t+1} + \lambda x_t + v_t$$
3. (x_t) output gap, (π_t) inflation rate, $(E_t\pi_{t+1})$ expected value at t of inflation at $t+1$, (μ_t) an exogenous demand shock, (v_t) an exogenous supply shock.

The New IS function

The IS function in three steps

1. You may find a very ellaborate way of deriving the IS function
2. **T**here is a simple and intuitive way to derive it
    1. Firstly, get the Euler equation
    2. Second, log-linearize the Euler equation
    3. Third, express the Euler equation in terms of percentage deviations from the steady state

The Marginal Rate of Substitution (MRS)

1. Assume a two period economy, with the household's objective of maximizing intertemporal utility from consumption $U(C_0, C_1)$

$$\max_{C_0, C_1} u(C_0) + \beta u(C_1)$$

2. The marginal rate of substitution (MRS) of intertemporal consumption is given by

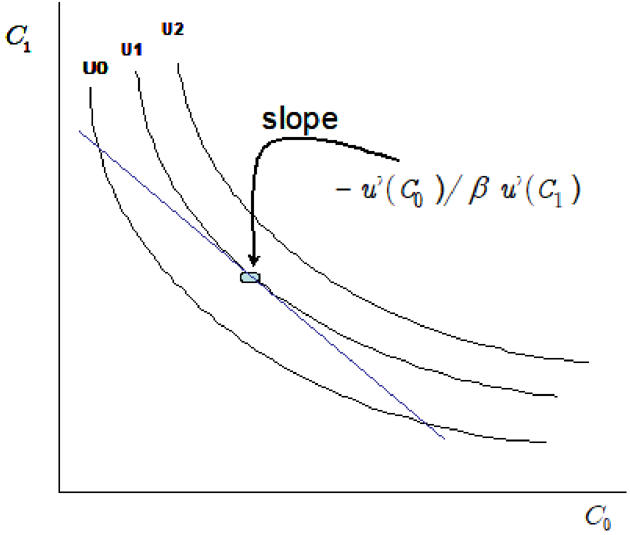
$$\begin{aligned} dU(C_0, C_1) &= 0 \\ u'(C_0) \cdot dC_0 + \beta u'(C_1) \cdot dC_1 &= 0 \end{aligned}$$

3. From where we can get

$$MRS = \frac{dC_1}{dC_0} = -\frac{u'(C_0)}{\beta \cdot u'(C_1)}$$

4. See next **Figure**

The utility trade-off between current vs future consumption



The Relative Price of intertemporal consumption

1. The households budget constraints in each period is given by

$$\begin{aligned} P_0C_0 + A_0 &= W_0 \\ P_1C_1 &= W_1 + A_0(1+r_0) \end{aligned}$$

   $P_{(\cdot)}$ is the price of consumption goods, $W_{(\cdot)}$ is wage income and savings (A_0) are invested in period 0.

2. The two constraints can be consolidated by cancelling out A_0

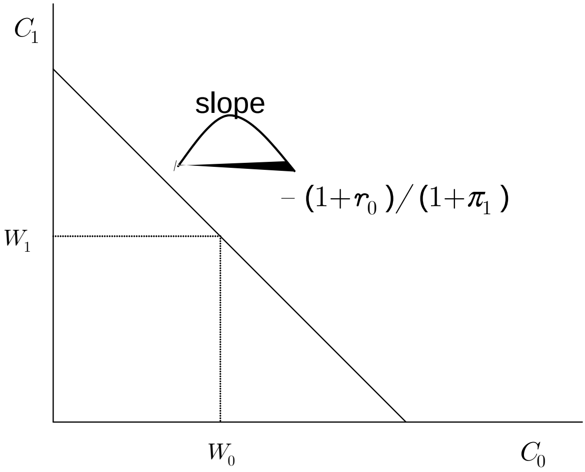
$$P_1C_1 = W_1 + (W_0 - P_0C_0)(1+r_0)$$

3. We can obtain the relative price (RP) of future consumption in terms current consumption (see next **figure)**.

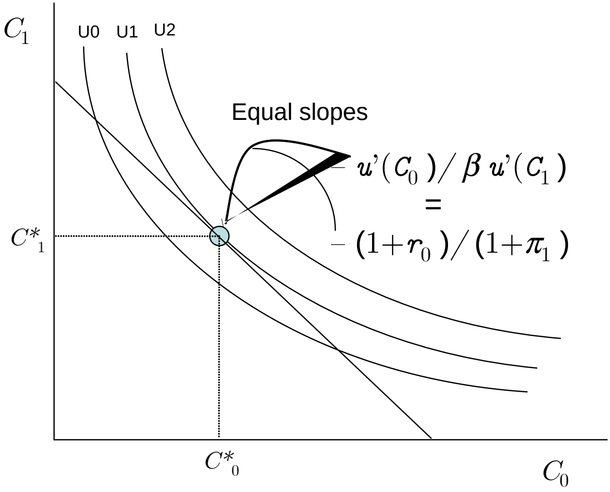
$$RP = \frac{dC_1}{dC_0} = -\frac{P_0}{P_1}(1+r_0) = -\frac{(1+r_0)}{(1+\pi_1)}$$

4. π_1 is the rate of inflation between t_0 and t_1; $P_1 = (1+\pi_1)P_0$.

The costs of current vs future consumption



The maximization of intertemporal utility: graphically



The Euler equation

1. Therefore, the maximization of utility is given by the condition

$$
\begin{aligned}
MRS &= RP \\
-\frac{u'(C_0)}{\beta \cdot u(C_1)} &= -\frac{(1+r_0)}{(1+\pi_1)} \\
u'(C_0) &= \beta \cdot \left[u'(C_1) \frac{(1+r_0)}{(1+\pi_1)} \right] \qquad (1)
\end{aligned}
$$

2. Eq. (1) is the **Euler equation,** already known from previous materials.
3. It relates current to future consumption in an optimal manner
4. And as we will show, it is also the **basis for the new IS function**

The Euler equation with uncertainty

1. Until now, we have assumed **no uncertainty** about the future level of consumption (C_1) and inflation (π_1)
2. **With uncertainty** about C_1 and π_1, we have just to add an expectations operator (E_0),
3. Now, the Euler equation looks like

$$u'(C_0) = \beta \cdot E_0 \left[u'(C_1) \frac{(1 + r_0)}{(1 + \pi_1)} \right]$$

4. Intuition 1: $\uparrow r_0 \Rightarrow \uparrow u'(C_0) \Rightarrow \downarrow C_0$
5. Intuition2: $\uparrow E_0 \pi_1 \Rightarrow \downarrow u'(C_0) \Rightarrow \uparrow C_0$
6. Why the term on the left hand side has no expectations operator attached?

Log-linearize the Euler equation

1. To further simplify the presentation, assume that the utility function is CRRA for $t = 0, 1, ...$:

$$U(c_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \Rightarrow u'(C_t) = C_t^{-\sigma}$$

2. With this, the Euler equation can be written as

$$C_t^{-\sigma} = \beta \cdot E_t \left[C_{t+1}^{-\sigma} \frac{(1+r_t)}{(1+\pi_{t+1})} \right]$$

3. Apply logs to the previous equation

$$\begin{aligned} -\sigma \ln C_t &= \ln \beta + \ln \left(E_t \left[C_{t+1}^{-\sigma} \frac{(1+r_t)}{(1+\pi_{t+1})} \right] \right) && (2) \\ &= \ln \beta + \ln E_t \left[\frac{(1+r_t)}{(1+\pi_{t+1})} \right] - \sigma \ln E_t C_{t+1} && (3) \end{aligned}$$

Log-linearize the Euler equation (continued)

1. We know that for a small constant ξ, $(-1 < \xi < 1)$, we get
$$\ln(1+\xi) \approx \xi$$
2. Applying the expectations operator, implies that
$$E\ln(1+\xi) \approx E(\xi) \approx \ln E(1+\xi)$$
3. Therefore, the second term on the right hand-side of equation (3) can be written
$$\ln E_t\left[\frac{(1+r_t)}{(1+\pi_{t+1})}\right] \approx E_t\left[\ln(1+r_t) - \ln(1+\pi_{t+1})\right] \quad (4)$$
$$\approx r_t - E_t\pi_{t+1} \quad (5)$$
4. and equation (3) will come as
$$-\sigma\ln C_t = \underbrace{\ln\beta}_{\approx 0} + (r_t - E_t\pi_{t+1}) - \sigma\ln E_tC_{t+1} \quad (6)$$
5. Notice: as $(1+r_t)$ is known at t, no expectations operator here.

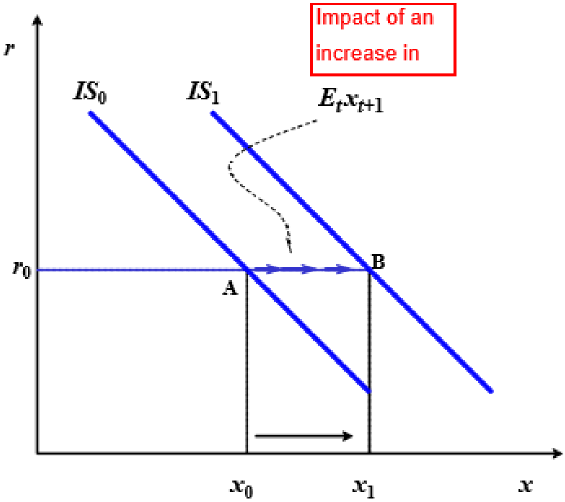
The Euler equation as % deviations from the steady state

1. To simplify exposition, let's use **small letters** to express variables in **log values**, that is
$$c_t = \ln C_t$$
2. Therefore, eq. (6) can be written as
$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) \tag{7}$$
3. Assume for simplicity: **no investment** in the economy (capital remains constant over the short run), **no government expenditures**
4. Then the log of consumption is equal to the log of output
$$c_t = y_t$$
5. The linearized Euler equation (7) can be written as
$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) \tag{8}$$

The Euler eq. as % deviations from the steady state (II)

1. Let's define the output gap (x_t) as the difference between the log level of output (y_t) and the log level of potential output, or the steady state level, $(\bar{y})$
$$x_t = y_t - \bar{y}$$
2. After some rearrangement, eq. (8) can be rewritten as
$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) + \mu_t \qquad (9)$$
3. where we added μ_t as exogenous demand shocks: $\mu_t = \rho_\mu \mu_{t-1} + \epsilon_t$, with $\epsilon_t \sim (0, \sigma_\epsilon^2)$ and $0 < \rho_\mu < 1$.
4. If families expect future output gap to increase $(\uparrow E_t x_{t+1})$, current demand increases and the current output gap will increase $(\uparrow x_t)$
5. If expected future inflation increases $(\uparrow E_t \pi_{t+1})$ more than the increase in current interest rates $(\uparrow r_t)$, the increase in r_t does not constrain the level of current economic activity $(\uparrow x_t)$. **See Fig.5.**

The New IS function



The New AS function (or the New Keynesian Phillips Curve)

1. As shown in the opening slides, the New AS function, or the New Keynesian Phillips curve (NKPC),can be written as

$$\pi_t = \beta \cdot E_t \pi_{t+1} + \lambda x_t + v_t \tag{10}$$

2. where v_t is a supply shock, $v_t = \rho_v v_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim (0, \sigma_v^2)$, and $0 < \rho_v < 1$.
3. To derive the New AS function can be a hard task
4. But as we did for the IS function, there is also an **intuitive way**: let's follow the latter to derive the NKPC
5. Be aware that there are **more formal and sophisticated** ways to get there, and these are required if you want to know all details, but in the end you get to the same results, so for now this suffices ...

Assumptions of the New AS function

1. **The crucial part of the model**: nominal rigidity known as **"Calvo Pricing"**, with four main assumptions:
2. **A1**. In each period there is a proportion of firms that **do not reset their prices**: θ.
3. **A2**. There is monopolistic competition in the goods market: firms set prices (the **frictionless price**, p_t^*) with a markup (ℓ) over marginal costs (mc_t). In logs we have

$$p_t^* = \ell + mc_t \tag{11}$$

4. **A3**. Because firms know that the price they set today remain constant until some time in the future, they **set prices today** (z_t) to minimize the loss in profits for not resetting their prices until then
5. **A4**. **Real marginal costs** in logs $(mc_t - p_t)$ are **a linear function of the output gap** in logs (x_t)

$$mc_t - p_t = \gamma x_t$$

Minimizing the Loss function

1. Given those assumptions, the **Loss function** $L(z_t)$ is given by

$$L(z_t) = \sum_{n=0}^{\infty} (\theta\beta)^n \cdot \underbrace{E_t \left(z_t - p^*_{t+n}\right)^2}_{\text{expected loss in profits}} \tag{12}$$

2. where $0 < \beta < 1$ is a discount factor, and z_t is the log price that minimizes the loss of profits due to no change in prices until $t+n$
3. Notice: θ^n is the probability of having the price fixed until $t+n$
4. To minimize $L(z_t)$ with respect to z_t, we set

$$\frac{\partial L}{\partial z_t} = 0$$

Minimizing the Loss function (continued)

1. To minimize $L(z_t) = \sum_{n=0}^{\infty} (\theta\beta)^n \cdot E_t \left(z_t - p^*_{t+n}\right)^2$ wrt z_t, we get

$$\begin{aligned}
\frac{\partial L}{\partial z_t} &= 0 \\
2 \sum_{n=0}^{\infty} (\theta\beta)^n \cdot E_t \left(z_t - p^*_{t+n}\right) &= 0 \qquad (13) \\
\underbrace{\sum_{n=0}^{\infty} (\theta\beta)^n \cdot z_t}_{=\frac{1}{1-\theta\beta} \cdot z_t} &= \sum_{n=0}^{\infty} (\theta\beta)^n \cdot E_t p^*_{t+n} \\
z_t &= (1 - \theta\beta) \sum_{n=0}^{\infty} (\theta\beta)^n \cdot E_t p^*_{t+n} \qquad (14)
\end{aligned}$$

2. The optimal price set by firms (z_t) is a weighted average of the prices that would be set in the future if there were no price rigidities

Minimizing the Loss function (continued)

1. We can use eq (11) to substitute for p_t^*

$$\begin{aligned} p_t^* &= \ell + mc_t \\ z_t &= (1-\theta\beta)\sum_{n=0}^{\infty}(\theta\beta)^n \cdot E_t p_{t+n}^* \end{aligned}$$

2. The result comes as

$$z_t = (1-\theta\beta)\sum_{n=0}^{\infty}(\theta\beta)^n \cdot E_t\left(\ell + mc_{t+n}\right) \tag{15}$$

3. Once we have z_t, it is easy to obtain the level of the **aggregate price level** (p_t)

$$\begin{aligned} p_t &= \theta p_{t-1} + (1-\theta)z_t \\ & \quad or \\ z_t &= \frac{1}{1-\theta}\left(p_t - \theta p_{t-1}\right) \end{aligned} \tag{16}$$

4. p_{t-1} is last period's aggregate price level, z_t is the new reset price

Minimizing the Loss function (continued)

1. In order to continue we have to **apply a trick**. Remember from the "Solution to Rational Expectations Models" that a sequence like

$$y_t = a \sum_{n=0}^{\infty} b^n \cdot E_t x_{t+n}$$

2. ... is a solution to the first order stochastic difference equation if $|b| < 1$

$$y_t = a x_t + b E_t y_{t+1}$$

3. Applying this reasoning to (15), this eq. can be re-written **(vide appendix)**

$$z_t = \theta\beta \cdot E_t z_{t+1} + (1 - \theta\beta)(\ell + mc_t) \tag{17}$$

4. So equating both eq. (16) and (17), we get

$$\frac{1}{1-\theta}(p_t - \theta p_{t-1}) = \theta\beta \cdot \frac{1}{1-\theta}(E_t p_{t+1} - \theta p_t) + (1 - \theta\beta)(\ell + mc_t) \tag{18}$$

5. See **next slide for details**

Minimizing the Loss function (continued)

1. Eq. (16) was given by

$$z_t = \frac{1}{1-\theta}\left(p_t - \theta p_{t-1}\right)$$

2. Eq.(15) was given by

$$z_t = \theta\beta \cdot E_t z_{t+1} + (1-\theta\beta)\left(\ell + mc_t\right)$$

3. Equating both we get

$$\frac{1}{1-\theta}\left(p_t - \theta p_{t-1}\right) = \theta\beta \cdot \frac{1}{1-\theta}\left(E_t p_{t+1} - \theta p_t\right) + (1-\theta\beta)\left(\ell + mc_t\right)$$

4. Leading to, after some rearrangement

$$\underbrace{\pi_t}_{=p_t - p_{t-1}} = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\ell + \underbrace{mc_t - p_t}_{\text{real marg.cost}}\right)$$

Minimizing the Loss function (continued)

1. Therefore, the New Phillips Curve can be written

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(\ell + \underbrace{mc_t - p_t}_{\text{real marg.cost}})$$

2. **Now the final step**. According to Assumption 4 real marginal costs are a log linear function of the output gap (x_t)

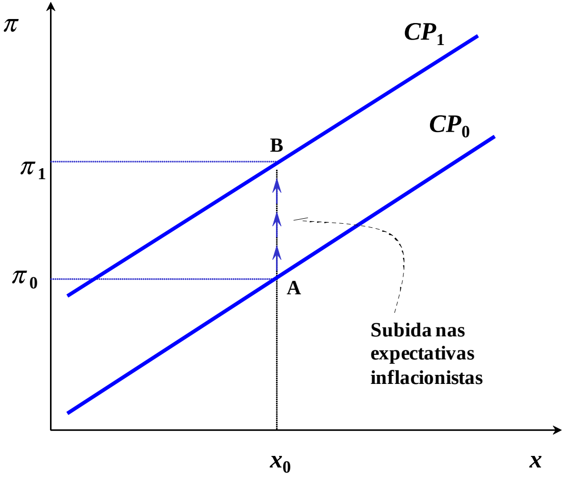
$$mc_t - p_t = \gamma x_t$$

3. Therefore, the AS function can finally come out as

$$\pi_t = \beta \cdot E_t \pi_{t+1} + \lambda x_t$$

4. with $\lambda = \dfrac{\gamma(1-\theta)(1-\theta\beta)}{\theta}$. Add technological shocks $v_t = \rho_v v_{t-1} + \varepsilon_t$, and there you have: eq (10).

The New Phillips Curve



The Central Bank behavior

The Central Bank behavior

1. Until now, we have 3 endogenous variables $\{x_{t+s}, \pi_{t+s}, r_{t+s}\}_{s=0}^{s=\infty}$, but only two equations (IS, AS or the New Phillips curve).
2. So we need another equation in order to have the model determined.
3. There are two major ways to close the model:
    1. The central bank controls the money supply, and the market determines r_t, (the **old view**)
    2. The central bank controls the r_t, and the market determines the level of money in the market (the **new view**).
4. Next: what is better, the old view or the **new view**.

Appendix

Appendix: explaining how eq. (21) is obtained from (19)

if $|b| < 1$ and $(\theta\beta) < 1$, then

$$\begin{aligned} y_t &= ax_t + bE_t y_{t+1} \\ &\downarrow \\ solution\ is &: \\ y_t &= a\sum_{n=0}^{\infty} b^n \cdot E_t x_{t+n} \end{aligned}$$

$$\begin{aligned} z_t &= (1-\theta\beta)\sum_{n=0}^{\infty}(\theta\beta)^n \cdot E_t\left(\ell + mc_{t+n}\right) \\ &\downarrow \\ solution\ to &: \\ z_t &= \theta\beta \cdot E_t z_{t+1} + (1-\theta\beta)\left(\ell + mc_t\right) \end{aligned}$$