6. Monetary policy conduct in "New Keynesian" settings: Credibility problems (II)*

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Abstract

Notes for the course "Monetary Economics: Macro Aspects," Spring 2006. The relevant literature behind these notes is:

Clarida et al. (1999).

Recommended reading (not required): Clarida et al. (2000); Woodford (1999), Walsh (2003, Chapter 5, 230-254).

1 Introductory remarks

- Recent research since the mid 1990s on monetary policymaking has focused on models with micro-foundations and incomplete nominal adjustment. The first building bloc is necessary to avoid the Lucas critique when comparing various policy regimes, and the second seems necessary if monetary policy models should have explanatory power at short-run frequencies. The focus has mostly been on models with the nominal interest rate as instrument (as is the operating target of most real-life central banks these days).
- This growing body of research has built a bridge between academics and practitioners
  - It uses models that academics can "accept" (both theoretically and — to various extent, of course — empirically)
  - It is empirically oriented, and are formulated in ways that real-life central banks appreciate
- The research has had enormous influence in recent focus (or return) on the importance of monetary policymaking for business cycle fluctuations. New issues are discovered/developed and old results re-emerge in new settings. Thus one witnesses a healthy mix of progress and confirmation in this area (i.e., scientific development).

2 The modern "New Keynesian" model of monetary policy analysis

- We start with a presentation of simplest, and widely applied, variant of a micro-founded, small-scale macro model. Some label models of this type as belonging to the "New Neoclassical Synthesis."
  - "Neoclassical" due to the reliance on private sector optimization
  - "New" reflecting the underlying assumptions of no instantaneous market clearing
- Clarida et al. (1999) denote the models "New Keynesian"
  - "Keynesian" as the frictions providing a role for monetary policy is usually nominal rigidities
  - "New" as the models, in contrast with "old" Keynesian models, are derived from first principles, i.e., are micro founded.
- This version of model is an "IS/AS" variant of great simplicity. But despite simplicity, its "yield" in terms of understanding monetary policy problems is far beyond the sum of its (two) parts.

The demand side

- The demand side originates from the expression guiding intertemporal consumption choice, which are a cornerstone of neoclassical analysis. The Keynes-Ramsey rule in log-deviations from steady state is given by:

$$c_t = \mathrm{E}_t c_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right), \qquad \varphi > 0.$$

Market clearing (in levels) is

$$Y_t = C_t + G_t$$

with G_t being exogenous, potential stochastic, government spending (it need not be government spending; it may broadly reflect any non-interest rate elastic demand component). This equation is rewritten as

$$1 = \frac{C_t}{Y_t} + \frac{G_t}{Y_t}.$$

In logs one therefore gets:

$$\begin{aligned} c_t &= y_t - e_t, \\ e_t &\equiv -\ln\left(1 - \frac{G_t}{Y_t}\right). \end{aligned}$$

Using this, the Keynes-Ramsey rule can be stated in terms of output (in log-deviations from steady state):

$$y_t = \mathrm{E}_t y_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + e_t - \mathrm{E}_t e_{t+1}.$$

Now, define the *output gap* as

$$x_t \equiv y_t - z_t$$

where z_t is stochastic, "hypothetical," flex-price output; the *natural rate of output.* Note that z_t can be interpreted as a technology shock (determining output fluctuations in a Real Business Cycle model). The "IS curve" then provides a dynamic relation for the output gap:

$$x_t = \mathrm{E}_t x_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + e_t - \mathrm{E}_t e_{t+1} - z_t + \mathrm{E}_t z_{t+1}$$

or simpler,

$$x_t = \mathrm{E}_t x_{t+1} - \varphi\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + g_t$$

where

$$\begin{aligned} g_t &\equiv \Delta \mathrm{E}_t z_{t+1} - \Delta \mathrm{E}_t e_{t+1}, \\ g_t &= \mu g_{t-1} + \widehat{g}_t, \qquad 0 < \mu < 1. \end{aligned}$$

Note that although it is considered as a demand-side relationship, it contains supply-side elements through z_t. I.e., g_t is not a pure demand shock

- $g_t < 0$ could be current below-average government expenditures driving output below natural
- $g_t < 0$ could be current above-average technology, causing output to be below the natural rate

The supply side

- There is monopolistic competition in goods markets. Prices are inflexible, and it is assumed that prices are set according to a Taylor-Calvo-style staggered price setting scheme (in style of Chapter 5.3.2 in Walsh, 2003). In particular, the so-called Calvo set-up, has the following features:
  - In each period any firm faces a state-independent probability of "being stuck" with its price, $0 < \theta < 1$
  - The probability is independent of when the firm last changed its price
  - Stylistic (and unrealistic?) representation of staggering. The independence of history facilitates aggregation:
    - ∗ θ is the fraction of firms not adjusting prices in a period
    - ∗ $1 - \theta$ is the fraction of firms adjusting is a period
    - ∗ $1 + \theta + \theta^2 + \theta^3 + ... = 1/(1 - \theta)$ is average duration of a price contract
- When "allowed" to set prices "today" the firms maximize the present value of current and expected future real profits. Hence, expectations about future prices become of importance (as in simple Taylor two-period staggering)
- Log-linearized, aggregate optimal price setting is characterized by the following inflation equation:

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + u_t, \qquad \lambda > 0, \quad 0 < \beta < 1, \qquad (2.2) \\ u_t &= \rho u_{t-1} + \widehat{u}_t, \qquad 0 < \rho < 1 \end{aligned}$$

  - Aggregate prices today, depend on prices yesterday (as prices are sticky)
  - Aggregate prices today, depend on aggregate prices for tomorrow
  - Aggregate prices today are a mark-up over real marginal costs; here proportional to the output gap
  - λ is an (inverse) measure of nominal rigidity in the economy: High θ means low λ.

    * In the limit: $\theta \to 0$, $\lambda \to \infty$ and prices are fully flexible, and $x_t = 0$; i.e., $y_t = z_t$.

  - Shock u_t captures variations in prices not captured by output gap (e.g., fluctuations in firms' mark up)

  Note that (2.2) is an expectations-augmented Phillips curve

- Taken together, the IS and AS curve depicts a simple monetary transmission mechanism as in earlier models under interest rate operating procedures:

  - The short nominal interest rate affects the real interest rate and aggregate demand and thus the output gap
  - Inflation is then affected by the output gap

- Note that both the IS curve and the Phillips curve are forward looking. Hence, current values of x_t and π_t depend on their expected future values, and thus expected future monetary policy. Indeed, forwarding the IS curve successively yields

$$x_t = \mathrm{E}_t \sum_{i=0}^{\infty} \{-\varphi (i_{t+i} - \pi_{t+1+i}) + g_{t+i}\} . \tag{2.5}$$

  I.e., current output gap is determined by sum of current and expected future nominal interest rates. Under the expectations theory of the term structure: Current output gap depends on the long real interest rate.

- Obviously, credibility of announcements about future policies will be important for macroeconomic performance

2.1 Stability properties

- Before examining monetary policy within the model, we look into the stability properties of the economic system in absence of shocks. Note that there are *no* predetermined state variables in the model. Both x_t and π_t are endogenous, and we require that the system of expectational difference equations

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} + \varphi \mathrm{E}_t \pi_{t+1} && (*) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t && (**) \end{aligned}$$

  provides unique, non-explosive, solutions for x_t and π_t. To analyze stability properties, one usually formulates the system in matrix form:

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}$$

where $\mathbf{A}$ is a 2×2 matrix. Uniqueness of non-explosive solutions requires that the real parts of the eigenvalues — the characteristic roots — of $\mathbf{A}$ are both numerically greater than one as system contains two "jump variables" x_t, and π_t. I.e., we require two "unstable" eigenvalues (cf. Blanchard and Kahn, 1980, *Econometrica*). The intuition is that with two unstable eigenvalues, *any* deviation from fundamentals-based solution lead to explosive paths. Only, $x_t = \pi_t = 0$ will be a non-explosive solution. In the case where exogenous shocks are re-introduced, one has that any movement in x_t and π_t will be induced by these shocks only.

- Remark the difference from previous analyses in a model with two predetermined variables. There, stability required two stable eigenvalues. Otherwise, the system would for any history, explode. But in the present model, there are no predetermined variable, only forward-looking variables. Had system instead been characterized by one predetermined variable and one jump variable, uniqueness of a non-explosive rational expectations equilibrium would require one stable and one unstable eigenvalue; one would then have a "saddle-path" equilibrium.[1]

- We now analyze $\mathbf{A}$. From (**) and (*) we get

$$\mathrm{E}_t x_{t+1} = x_t - \varphi\beta^{-1}\left[\pi_t - \lambda x_t\right]$$

$$\mathrm{E}_t x_{t+1} = \left(1 + \varphi\beta^{-1}\right) x_t - \varphi\beta^{-1}\pi_t$$

With $\mathrm{E}_t \pi_{t+1} = \beta^{-1}\left[\pi_t - \lambda x_t\right]$ from (**), we get the system as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \varphi\lambda\beta^{-1} & -\varphi\beta^{-1} \\ -\lambda\beta^{-1} & \beta^{-1} \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}$$

The eigenvalues δ_1 and δ_2 of $\mathbf{A}$ are computed from:

$$\begin{vmatrix} 1 + \varphi\lambda\beta^{-1} - \delta & -\varphi\beta^{-1} \\ -\lambda\beta^{-1} & \beta^{-1} - \delta \end{vmatrix} = 0.$$

This gives a second-order polynomial in δ:

$$\delta^2 - \beta^{-1}\left[1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right]\delta + \beta^{-1} = 0,$$

$$\beta\delta^2 - \left[1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right]\delta + 1 = 0,$$

where the solutions are:

$$\delta = \frac{1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right) \pm \sqrt{\left(1 + \beta^{-1}\left(1 + \varphi\lambda\beta^{-1}\right)\right)^2 - 4\beta}}{2\beta}$$

We now make a numerical check on these values. Assume the following parameters:

[1] Note the analogy with the standard Ramsey growth model formulated in consumption (the jump variable) and capital (the predetermined variable). In that model, the unique non-explosive equilibrium requires that the system has a stable and unstable eigenvalue (in continuous time: one below zero and one above zero; in discrete time, one below and one numerically above one)

- Aggregate demand's real interest rate sensitivity; $\varphi = 0.1$
- Inflation's sensitivity to aggregate demand; $\lambda = 0.14$
- Discount factor; $\beta = 0.99$

With these values we get

$$\delta_1 = 0.8347$$
$$\delta_2 = 1.2101$$

Hence, the system does **not** provide unique solutions for x_t and π_t! For a fixed nominal interest rate, the economy will feature **infinitely many** non-explosive output and inflation paths. For an arbitrary value of x_t there is a value of π_t consistent with rational expectations, and the economy will converge to $x_t = \pi_t = 0$ over time.[2] There is no unique rational expectations equilibrium. This is known as indeterminacy. (Note: the result does not rely on the particular parameter values; it can be proven generally.) Hence, the price level indeterminacy under interest rate operating procedure of earlier models is replaced by inflation and output gap indeterminacy — **real indeterminacy**

- Intuition for this indeterminacy: An **arbitrary** increase in inflation expectations will — for a given nominal interest rate — **decrease** the real interest rate, and **increase output** and **increase inflation**
- Self-fulfilling prophecy!
- The economy will gradually return to steady state following this "sun-spot driven" burst of inflation and output
- The economy may thus be subject to fluctuations in output and inflation that has nothing to do with economic shocks (here g_t and u_t); i.e., there may be non-fundamental driven fluctuations

• The purpose of the deriving guidelines for good monetary policy is therefore again two-fold:

  - Secure that the economy will not be subject to self-fulfilling bursts of inflation and output
    * I.e., secure a determinate — unique — equilibrium for inflation and output gap
  - Secure the optimal manner by which the output gap and inflation fluctuates in response to fundamental shocks

[2] Had x_t been a predetermined variable, this would have been just fine. The model would be saddle-path stable, and would have given a unique value of π_t given this value of x_t. But x_t is *endogenous*, and needs to be "pinned down" by the model. Here, we have a saddle path in a two-dimensional space, but two free variables to place *anywhere* on that path!

3 Optimal monetary policy under discretion

- The criterion of monetary policy is to minimize the expected discounted sum of deviations of output gap and inflation from their long-run equilibrium values. In each period t, the **utility function** is assumed to be

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2, \qquad \alpha > 0.$$

  Recent research has shown that such a function can be derived as a second-order Taylor approximation of the utility function of the representative agent in the economy. (See Walsh, 2003, Chapter 11, but this is only supplementary reading). Note a zero inflation target. This can be motivated by adherence to the underlying monopolistic competition and price rigidity: Any inflation rate means that some prices are not optimally adjusted (due to price stickiness), and this results is inefficient dispersion in production of the various goods in the economy. Note that the output target equals the natural rate (this may not be reasonable under monopolistic competition, as output even under flexible prices would be inefficient — see later)

- Due to forward looking nature of model, there is difference between solution under commitment to a policy path, or period-by-period optimization (discretion). We examine discretion here. To solve model note first:

  - In period t, policy cannot affect expectations about future variables (no persistence in equations, and no commitment is assumed possible)
  - Hence, when optimizing, expected future variables are taken as given

  Mathematical trick: Treat x_t as the policy instrument, and find i_t compatible with the solution afterwards. Maximizing

$$-\frac{1}{2}\mathrm{E}_t \sum_{i=1}^{\infty} \beta^i \left[\alpha x_t^2 + \pi_t^2\right], \qquad 0 < \beta < 1$$

  w.r.t. i_t subject to (2.1) and (2.2) is thus equivalent of maximizing

$$-\frac{\alpha}{2}x_t^2 - \frac{1}{2}\pi_t^2 + F_t \tag{3.1}$$

  w.r.t. x_t subject to

$$\pi_t = \lambda x_t + f_t \tag{3.2}$$

  taking **as given** F_t and f_t

- The problem becomes a sequence of single-period problems

- Simple first-order conditions:

$$-\alpha x_t = \lambda \pi_t \tag{3.3'}$$

These depicts a so-called "lean against the wind" policy: If inflationary pressures arise ($\pi_t > 0$), contract output ($x_t < 0$) such that the marginal cost (left-hand side) equals the marginal gain (the right hand side). Note that with more nominal rigidity, lower λ, the inflation-output trade-off is more unfavorable: A given reduction in output reduces inflation by less (in different wording: disinflation is more painful in terms of lost output with a flatter Phillips curve). Use the first-order condition in the Phillips curve to eliminate the output gap:

$$\begin{aligned}
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + u_t \\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} - \left(\lambda^2/\alpha\right) \pi_t + u_t \\
\pi_t &= \frac{\beta}{1+\lambda^2/\alpha} \mathrm{E}_t \pi_{t+1} + \frac{1}{1+\lambda^2/\alpha} u_t
\end{aligned}$$

This is a first-order expectational difference equation in π_t. Notice it has one unstable eigenvalue (when written as $\mathrm{E}_t \pi_{t+1} = \left(1+\lambda^2/\alpha\right)\beta^{-1}\pi_t - \beta^{-1}u_t$ one sees that $\left(1+\lambda^2/\alpha\right)\beta^{-1} > 1$) securing a unique non-explosive solution for inflation. Whenever policy ensures that (3.3') is satisfied, x_t is unique as well. This is solved for inflation by the method of undetermined coefficients. Conjecture a solution

$$\pi_t = X u_t$$

Forward the conjecture and take expectations to get:

$$\mathrm{E}_t \pi_{t+1} = X \mathrm{E}_t u_{t+1} = X \rho u_t.$$

Inserted back into difference equation:

$$X u_t = \frac{\beta}{1+\lambda^2/\alpha} X \rho u_t + \frac{1}{1+\lambda^2/\alpha} u_t.$$

This identifies X, as it must hold for any u_t, by

$$X = \frac{\beta\rho}{1+\lambda^2/\alpha} X + \frac{1}{1+\lambda^2/\alpha}$$

Hence,

$$X = \frac{1}{1+\lambda^2/\alpha - \beta\rho}$$

Solution for inflation then follows as

$$\begin{aligned}
\pi_t &= \frac{1}{1+\lambda^2/\alpha - \beta\rho} u_t \\
&= \alpha \frac{1}{\lambda^2 + \alpha\left(1-\beta\rho\right)} u_t,
\end{aligned} \tag{3.5}$$

and the solution for output gap follows from the first-order condition as

$$x_t = -\lambda \frac{1}{\lambda^2 + \alpha(1-\beta\rho)} u_t \tag{3.4}$$

- Implications:
  - No impact of demand and technology shocks; these pose no trade-offs. If demand gets higher than the natural rate, policy contracts to bring demand back such that output is at the natural rate. If the economy experiences technological advancement, monetary policy loosens to bring output up in accordance with the natural rate.[3]
  - Impact of a "cost-push" shock is "spread out" on inflation and output gap; i.e., there *is* a trade-off in monetary policy. Monetary policy contracts (induces a negative output gap, $y_t < z_t$) to dampen inflation, but not completely.

- Solution for x_t and π_t and the associated solution for $\mathrm{E}_t x_{t+1}$ and $\mathrm{E}_t \pi_{t+1}$ (e.g., $\mathrm{E}_t \pi_{t+1} = \rho \pi_t$) can be used in IS-curve to find associated solution for the nominal interest rate. The optimal value of the nominal interest rate can be written in many ways. If written as a function of expected next-period inflation one gets:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \varphi(i_t - \mathrm{E}_t \pi_{t+1}) + g_t \qquad (2.1) \\ -\frac{\lambda}{\alpha}\pi_t &= -\frac{\lambda}{\alpha}\mathrm{E}_t \pi_{t+1} - \varphi(i_t - \mathrm{E}_t \pi_{t+1}) + g_t \\ -\frac{\lambda}{\alpha\rho}\mathrm{E}_t \pi_{t+1} &= -\frac{\lambda}{\alpha}\mathrm{E}_t \pi_{t+1} - \varphi(i_t - \mathrm{E}_t \pi_{t+1}) + g_t \end{aligned}$$

and thus

$$\begin{aligned} i_t &= \left(1 + \frac{1}{\varphi}\left[\frac{\lambda}{\alpha\rho} - \frac{\lambda}{\alpha}\right]\right)\mathrm{E}_t \pi_{t+1} - \frac{1}{\varphi} g_t \\ &= \left(1 + \frac{\lambda(1-\rho)}{\varphi\alpha\rho}\right)\mathrm{E}_t \pi_{t+1} - \frac{1}{\varphi} g_t \qquad (3.6) \end{aligned}$$

Hence, written like this, an increase in expected inflation is met by a *larger* increase in the nominal interest rate, i.e., the real interest rate increases

- Such an interest rate rule typically secures determinacy
  - Self-fulfilling burst of inflation and output increases are **ruled out**

[3] This is what many argue happened in the US in the 1990s boom. Output went up, nominal interest rates remained stable, yet there was no markedly inflation (as one would expect from traditional Keynesian analysis). Some observers saw this as the wake of a "New Economy" where growth was now non-inflationary as compared to the past. The simple model of the main text shows that one could "just" have witnessed smart monetary policy making in a boom caused by booming technology in an "Old Economy."

  - An arbitrary increase in inflation expectations increases the real interest rate, decreases output gap and inflation
  - ....invalidates the increase in inflation as a rational expectations equilibrium

- Empirical analysis of interest rate rules by Clarida, Galí and Gertler (2000, *Quarterly Journal of Economics*): For US, in 1970s the estimated coefficient on expected inflation was below one. In 1980s and onwards, the estimated coefficient is significantly above one

  Combining the empirical results with theory:

  - The high and persistent inflation rates in the 1970s could have been expectations driven; monetary policy did not respond sufficiently aggressive towards rising inflation expectations
  - In the 1980s and onwards (when Paul Volcker and later Alan Greenspan took office), there were less fluctuations and lower average inflation. This is consistent with the fact that the possibility of self-fulfilling inflationary bursts are ruled out

  Note, though, that these empirics are not "proof" of self-fulfilling fluctuations; but "only indicative evidence"

- Other implications of optimal monetary policy under discretion:

  - After a cost-push shock, inflation gradually moves back towards target; in accordance with inflation targeting (more on this later). Caveat: so does the output gap; so is it output gap targeting?
    * Note that we cannot tell from the an interest rate rule like (3.6), what is in the loss function of the central bank
    * (3.6) could look as if output gap did not enter; moreover, the size of the coefficient is not exclusively determined by preferences, i.e., α)
  - If technology shocks are random walk, $z_{t+1} = z_t + \widehat{z}_t$, then the nominal rate does **not** respond to technology shocks (as g_t is unaffected by z_t)
    * If $z_t > 0$ current output gap falls, but for given expected future output gap, expected future output increases and current output increases, leaving the output gap unchanged
    * Inflation does not change either => Increasing output, no inflation, no central bank response is compatible with optimality (cf. Footnote 3)

- Distinguishing the source of disturbances is therefore **very** important for monetary policy conduct

4 Rules versus discretion and credibility problems

- **Discretionary** solution in simple "New Keynesian" model is **suboptimal.** With forward-looking variables: differences between ex ante and ex post optimal policies. In this new type of models, the commitment policies — the ex ante optimal policies — have interesting features

  - They have features that confirm results from early credibility literature (the Barro and Gordon type models)
  - They have features that are new, and which requires new thinking about the optimal strategy for monetary policymaking

- "Resurrection" of the conservative central banker — for completely new reasons

4.1 Credibility problems and "Rogoff-conservatism"

- With forward-looking variables, credibility of monetary policy becomes important. Immediately evident if per-period loss function takes the form:

$$-\frac{\alpha}{2}(x_t - k)^2 - \frac{1}{2}\pi_t^2, \qquad k > 0.$$

I.e., the natural rate of output is not the target, but due to monopolistic competition, $z_t + k$ is target output (remember, $x_t = y_t - z_t$). The term $k > 0$ represents the permanent output loss due to imperfect competition. Under discretion, optimal policy will lead to (see Appendix A) :

$$x_t = -\lambda \frac{1}{\lambda^2 + \alpha(1 - \beta\rho)} u_t \tag{4.3'}$$

$$\pi_t = \frac{1}{1 - \beta + \lambda^2/\alpha}\lambda k + \alpha \frac{1}{\lambda^2 + \alpha(1 - \beta\rho)} u_t \tag{4.4'}$$

I.e., almost the same solution as the previous outcomes, but now with a Barro-Gordon-style inflation bias, $\lambda k / \left(1 - \beta + \lambda^2/\alpha\right) > 0$. The explanation for this is the same as in the standard Barro-Gordon model. A Rogoff-conservative central banker, with utility function $\alpha^c < \alpha$, can thus be an improvement — also for the standard reasons.

An important insight is, however, highlighted by New-Keynesian literature. Even when $k = 0$, there can be gains from commitment, and gains from appointing a conservative central banker

- Intuitive idea:

  - With forward-looking inflation expectations, a policy affecting inflation expectations appropriately, can help stabilize current inflation

- If $u_t > 0$, a commitment to fight future inflation, reduces inflation expectations and therefore current inflation
- A smaller current output contraction can then attain a higher reduction in current inflation. I.e., the commitment improves the inflation-output gap trade off
- Hence, conservatism (=commitment to fight future inflation) can improve shock stabilization. This is in complete contrast with Barro-Gordon model, where conservatism *distorted* shock stabilization

Example of policy commitment

- This simple example highlights the stabilization gains from commitment. It is important to note, however, that the policy considered here is **not** the unrestricted optimal commitment policy. It is merely a form of "constrained" commitment that highlights the improvement in the inflation-output gap trade off through conservatism
- Assume again for simplicity that output gap is policy instrument. Assume then commitment is to a policy rule of the form

$$x_t^c = -\omega u_t \tag{4.5}$$

(nominal interest rate will follow from IS-equation when inflation is solved for); $\omega > 0$ is to be chosen optimally. Inflation follows from the Phillips curve:

$$\begin{aligned} \pi_t^c &= \beta \mathrm{E}_t \pi_{t+1}^c + \lambda x_t^c + u_t \\ &= \beta \mathrm{E}_t \pi_{t+1}^c - \lambda \omega u_t + u_t \qquad (1) \end{aligned}$$

Solving forward:

$$\begin{aligned} \pi_t^c &= \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[-\lambda \omega u_{t+i} + u_{t+i} \right] \\ &= \frac{1 - \lambda \omega}{1 - \beta \rho} u_t \qquad (4.9) \end{aligned}$$

The improved inflation-output gap trade-off is now evident. Solution for inflation can be expressed as

$$\pi_t^c = \frac{\lambda}{1 - \beta \rho} x_t^c + \frac{1}{1 - \beta \rho} u_t$$

Note:

$$\frac{\mathrm{d}\pi_t^c}{\mathrm{d}x_t^c} = \frac{\lambda}{1 - \beta \rho}$$

Under discretion, $\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + u_t$, and $\mathrm{E}_t \pi_{t+1}$ is taken as given; hence:

$$\frac{\mathrm{d}\pi_t^d}{\mathrm{d}x_t^d} = \lambda$$

Result:

$$\frac{\mathrm{d}\pi_t^c}{\mathrm{d}x_t^c} > \frac{\mathrm{d}\pi_t^d}{\mathrm{d}x_t^d}.$$

I.e., a given change in the output gap has larger impact on inflation under commitment (said with other words: a disinflation will be less costly in terms of lost output). The intuition is the following. Under commitment, expected future inflation is affected => larger impact on current inflation of current inflation.

- What is optimal value of ω? It solves

$$\max_{\omega} -\mathrm{E}_t \frac{1}{2} \sum_{i=0}^{\infty} \beta^i \left[\alpha \left(x_{t+i}^c \right)^2 + \left(\pi_{t+i}^c \right)^2 \right].$$

Since both x_{t+i}^c and π_{t+i}^c are proportional to the exogenous cost-push shock, the optimal ω solves simply

$$\max_{\omega} -\frac{1}{2} \left[\alpha \left(x_t^c \right)^2 + \left(\pi_t^c \right)^2 \right]$$

subject to

$$\pi_t^c = \frac{\lambda}{1 - \beta\rho} x_t^c + \frac{1}{1 - \beta\rho} u_t$$

I.e., find

$$\max_{\omega} -\frac{1}{2} \left[\alpha \left(-\omega u_t \right)^2 + \left(\frac{1 - \lambda\omega}{1 - \beta\rho} u_t \right)^2 \right]$$

The relevant first-order condition is

$$-\alpha\omega u_t^2 + \frac{1 - \lambda\omega}{1 - \beta\rho} u_t^2 \frac{\lambda}{1 - \beta\rho} = 0.$$

Alternatively,

$$\alpha x_t^c + \frac{\lambda}{1 - \beta\rho} \pi_t^c = 0$$

$$-\alpha^c x_t^c = \lambda \pi_t^c, \qquad \alpha^c \equiv \alpha \left(1 - \beta\rho \right) < \alpha$$

Remember that the first-order condition under discretion was:

$$-\alpha x_t = \lambda \pi_t$$

I.e., the precise same form, but under this form of commitment policy, $\alpha^c < \alpha$; i.e., Rogoff conservatism is beneficial.

- Example of benefits from commitment. By committing to being "tough" on inflation in the future, the inflation-output gap trade-off is improved.[4] But will this be credible? When the "future arrives," the central bank has incentive to act according to

$$-\alpha x_t = \lambda \pi_t$$

Hence, if this is believed by the private sector, expectations will not serve a role a an inflation stabilizing mechanism. So, the commitment policy is time-inconsistent. Appointing a conservative central banker could be remedy to (partially) resolve credibility problems. Just as in Barro and Gordon model, but for very different reasons.

- The example shows credibility issues in monetary policy, even when the target value of output is the natural rate, There is no average inflation bias, but a "stabilization bias" arises under discretionary policymaking. A bias that can be reduced if commitment is possible.

4.2 Characterization of the full commitment solution

- The example in the past section considered commitment to a particular form of policy rule ($x_t = -\omega u_t$). It was not the fully optimal commitment solution (but a "constrained optimum"). The "unconstrained" commitment solution has very interesting features, and we consider it now. The problem is to find a sequence of output gap and inflation that maximizes utility. The technique: Set up the Lagrange function:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}\mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha x_{t+i}^2 + \pi_{t+i}^2 \right.\right. \\ &\left.\left. +2\phi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+1+i} - \lambda x_{t+i} - u_{t+i} \right) \right] \right\} \end{aligned} \quad (4.17')$$

where $2\phi_{t+i}$ is the multiplier on the Phillips curve (we do not need the IS-curve as the nominal interest rate can be adjusted freely).[5] The relevant first-order conditions are

$$\alpha x_{t+i} - \lambda \phi_{t+i} = 0$$

$$\begin{aligned} \pi_{t+i} + \phi_{t+i} - \phi_{t+i-1} &= 0, \quad i > 0 \\ \pi_t + \phi_t &= 0, \end{aligned}$$

[4] To repeat, it is because future policy following an inflation shock will be more contractionary thereby reducing inflation expectations. For this channel to operate, it is necessary that the inflation shock has some persistence (otherwise future policies towards the shock is of no importance under this policy rule). Indeed, for $\rho = 0$ the optimal value of $\alpha^c = \alpha$ and no conservatism is optimal.

[5] Note that we cannot use dynamic programming here. Dynamic programming gives optimal policies at any period of time given the states of the economy at the given period. It thus gives time-consistent policies. And as just argued, these are not optimal (albeit credible).

In combination:

$$\begin{aligned} -\alpha\left(x_{t+i}-x_{t+i-1}\right) &= \lambda\pi_{t+i}, \quad i>0 \\ -\alpha x_t &= \lambda\pi_t \end{aligned}$$

Central implication: Commitment policy involves (for $i > 0$; i.e., all periods after t, but not in t)

$$\pi_{t+i} = -\frac{\alpha}{\lambda}\left(x_{t+i}-x_{t+i-1}\right) \qquad (4.18')$$

Hence, inflation and output gap exhibit history dependence. This is also known as optimal *policy inertia*. Again, the optimality of this arises from the fact that it will affect inflation expectations, and through this improve the inflation-output gap trade-off

- Intuition: Consider the simple case of $\rho = 0$ (only temporary inflation shocks — a case where the constrained commitment policy of the previous section has no merit, cf. Footnote 4) .
  - If $u_t > 0$, inflation rises, and optimal policy is contractive
  - With **policy inertia**, **next-period policy will also be contractive**
    - => Next-period inflation is dampened
    - => Current inflation expectations are dampened
    - => Current inflation is reduced
  - Thus a **mild**, but **prolonged** contraction provides better inflation stabilization

- Generally, policy inertia improves predictability of future policy, and current variables are easier to affect by smaller current policy adjustments. One may say that by adopting policy inertia, the central bank is "letting the market do some of the stabilization"

- Note again the inherent credibility problem of the commitment solution. When the temporary cost-push shock has "worn out," it is no longer optimal to contract policy. Typical difference between ex ante and ex post optimality and reflects the time-inconsistency of the commitment solution.[6] But if one cannot commit, one doesn't reap the gains in terms of better stabilization performance. Hence, institutional frameworks securing, or approaching, commitment policy is desirable. This therefore confirms the generality of the insights from the Barro and Gordon literature (and ultimately the foundation for the time-inconsistency literature, the Kydland and Prescott, 1977, paper).

[6] Technically, when we enter period $t+1$, it is optimal to follow $-\alpha x_{t+1} = \lambda\pi_{t+1}$ and $-\alpha\left(x_{t+i}-x_{t+i-1}\right) = \lambda\pi_{t+i}$, for $i > 1$. But this is a deviation from what was calculated as optimal in period t. I.e., the period-t plan is not time consistent.

5 Practical implications and extensions of simple model

- The main implications of the simple model presented in the past sections, carry over to more richer and realistic extensions
- **Imperfect information about current shocks**
  - Real world: Shocks g_t and u_t are not observed at the time of policy implementation
  - First-order condition for optimal policy (here discretion for simplicity) become one in expected terms:

$$-\alpha \mathrm{E}\left[x_t|\Omega_t\right] = \lambda \mathrm{E}\left[\pi_t|\Omega_t\right] \tag{5.1}$$

  - Solution the same as under perfect information, except for forecast errors
    - Note that forecast errors about g_t causes positive correlation between x_t and π_t (but fundamental-driven; not result of indeterminacy)
  - Scope for intermediate targeting, if intermediate targets exist that are
    - Readily observable
    - Correlated with goal variables
- **Instrument choice problem**
  - Imperfect information about shocks raises the issue about optimal operating procedure
  - Analogous to the Poole (1970) analysis
  - If model is extended with money market equilibrium and money demand shocks,
    - Interest targeting procedure is optimal if money demand shocks are predominant (and if money demand is rather interest rate insensitive)
- **Parameter uncertainty**
  - Uncertainty about the structural parameters of the economy
  - I.e., what is the true value of, e.g., φ?
  - Classic paper by Brainard (1967, *American Economic Review*) showed that such uncertainty usually called for "cautious" central bank behavior
    - Arises from the central bank's risk aversion: The loss from something going "wrong" by a policy choice is higher than the gain from when it is going "good"

- Evident to some extent by central banks' reluctance to move the nominal interest rate in large steps

- **Endogenous output and inflation persistence**
  - Empirically, it is hard to reject that
    a) Output and consumption depend on their own past values
    b) Inflation depends on its own past value
  - It is, however, controversial how strong this endogenous output and inflation persistence is
  - Introduction of such realistic persistence, does not affect qualitatively the results found in simple model
  - Output persistence is immaterial if effects of lagged output can be neutralized by the nominal interest rate
    * If interest variability is costless, output persistence may be unimportant
  - Inflation persistence, however, plays a crucial role:
    * Induces **worse** policy trade-off (current inflation is less "controllable") (more output costs of mitigating u_t shocks)
    * The "leaning against the wind" policy, however, is still optimal
    * All things equal, the convergence towards steady state will take longer time

- **Transmission lags**
  - Realistically, the nominal interest rate does no affect demand and inflation "simultaneously" as in simple model
  - Consensus is that demand is affected first, but with some lag; then inflation is affected by a further lag through the Phillips curve; cf. the VAR evidence. With one-period lags, this changes the optimality condition to

$$-\alpha \mathrm{E}_t x_{t+1} = \lambda \mathrm{E}_t \pi_{t+2} \tag{6.7}$$

  - Not much is qualitatively changed by this alteration; except that one can express the equilibrium nominal interest rate as a function of current output gap and one-period ahead inflation expectations; i.e., as a forward-looking Taylor rule
  - Note that g_t shocks then create a positive correlation between x_t and π_t as found in data

Appendix

A Finding the solution of the simple New Keynesian model when $k > 0$

This appendix shows how to derive equations (4.3') and (4.4'). The equations follow the format of Clarida et al. (1999), but in their paper they make a simplifying assumption by setting $\beta = 1$, which makes the constant term a very simple one. Clearly, $\beta = 1$ should then be applied in the coefficients to the u_t shock also, but I retain β explicitly, so that you could see that the coefficients on the shocks are the same as in the case of $k = 0$. Now, the central bank's per-period utility function is:

$$-\frac{\alpha}{2}\left(x_t - k\right)^2 - \frac{1}{2}\pi_t^2, \qquad k > 0,$$

and the AS curve is

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \lambda x_t + u_t.$$

The relevant first-order condition will be (insert the AS curve into the utility function, maximize w.r.t. x_t and take $\mathrm{E}_t\pi_{t+1}$ as given; just as in the case of $k = 0$):

$$-\alpha\left(x_t - k\right) = \lambda \pi_t.$$

This is inserted into the AS curve:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \lambda\left[\left(\lambda/\alpha\right)\pi_t - k\right] + u_t$$

$$\pi_t\left(1 + \lambda^2/\alpha\right) = \beta \mathrm{E}_t \pi_{t+1} + \lambda k + u_t$$

$$\pi_t = \frac{\beta}{1+\lambda^2/\alpha}\mathrm{E}_t\pi_{t+1} + \frac{\lambda}{1+\lambda^2/\alpha}k + \frac{1}{1+\lambda^2/\alpha}u_t$$

This is a first-order expectational difference equation with one unstable root (as in the case of $k = 0$); so there is a unique non-explosive solution to π_t. In the case with $k = 0$, it was conjectured that $\pi_t = Xu_t$, where X was the undetermined coefficient to be determined. Now, what makes the above difference equation different from the one with $k = 0$?

The presence of a constant term. Hence, a sensible conjecture is one that involves a constant term. I.e., conjecture a solution of the following format:

$$\pi_t = Y + Xu_t, \tag{*}$$

where Y and X are the undetermined coefficients to be determined.

Find the coefficients. Forward (*) one period, and take period t expectations:

$$\pi_{t+1} = Y + Xu_{t+1},$$

$$\mathrm{E}_t\pi_{t+1} = Y + X\mathrm{E}_t u_{t+1},$$

$$\mathrm{E}_t\pi_{t+1} = Y + X\rho u_t. \tag{**}$$

Insert this into the difference equation:

$$\pi_t = \frac{\beta}{1+\lambda^2/\alpha}\left[Y + X\rho u_t\right] + \frac{\lambda}{1+\lambda^2/\alpha}k + \frac{1}{1+\lambda^2/\alpha}u_t$$

$$\pi_t = \frac{\beta Y + \lambda k}{1+\lambda^2/\alpha} + \frac{X\beta\rho + 1}{1+\lambda^2/\alpha}u_t$$

This verifies the form of the conjecture, and identifies the coefficients by the equations

$$\begin{aligned} Y &= \frac{\beta Y + \lambda k}{1+\lambda^2/\alpha} \\ X &= \frac{X\beta\rho + 1}{1+\lambda^2/\alpha} \end{aligned}$$

These are solved to get

$$\begin{aligned} Y &= \frac{1}{1-\beta+\lambda^2/\alpha}\lambda k, \\ X &= \alpha\frac{1}{\lambda^2+\alpha\left(1-\beta\rho\right)}u_t \end{aligned}$$

Hence,

$$\pi_t = \frac{1}{1-\beta+\lambda^2/\alpha}\lambda k + \alpha\frac{1}{\lambda^2+\alpha\left(1-\beta\rho\right)}u_t,$$

which is equation (4.4) in Clarida et al. (1999) and (4.4') in the main text, when one simplifies the constant term by taking Clarida et al.'s simplifying assumption $\beta = 1$ into account (this makes $Y = \alpha/\lambda$).

B Key concepts you should know

A simple "New Keynesian" model of monetary policy analysis

- The intertemporal "IS-curve"
- The expectations-augmented "Phillips curve"
- The importance of forward-looking behavior
- For constant nominal interest rate, infinitely many non-explosive output and inflation paths (real indeterminacy)
- Purpose of monetary policy (nominal interest rate setting):
  - Minimize fluctuations in output gap and inflation
  - Secure a unique equilibrium for inflation and output gap

Optimal monetary policy under discretion

- The standard quadratic utility function in output gap and inflation
- The simple first-order condition for optimal policy
  - "Leaning against the wind" policy
- More nominal rigidity worsens the inflation-output gap trade off
- Characteristics of optimal policy outcomes
  - No effects of demand and technology shocks
  - The "cost-push" shock is spread out over output gap and inflation
- Characterization of associated interest rate setting
  - Formulated as function of expected future inflation, the nominal interest rate increases by more than an increase in inflation expectations => increases real interest rate => secures unique equilibrium
  - (Note that interest rate expression tells little about the preferences of the central bank.)

Optimal monetary policy under commitment

- Suboptimality of discretionary solution
- The case with positive target for output gap (inflation bias)
- The case with zero target for output gap (no inflation bias)
- Rogoff-conservatism as improvement over discretion
  - Signals future contractive behavior, which dampens current (forward-looking) inflation
  - Improves inflation-output gap trade-off and shock stabilization
- The stabilization bias of time-consistent monetary policy
- Fully optimal policy: The optimality of *inertial* policy
  - Inertia secures prolonged contractions following inflationary shocks
  - Improves inflation-output gap trade off