The Optimal Choice of Monetary Instruments — The Poole Model —

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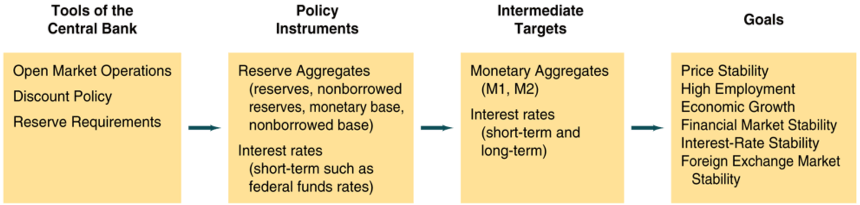
Summary

1. Tools, targets and goals
2. A simple model with demand shocks (the Poole model)
3. Extending the model: monetary base as potential instrument
4. Poole's model with supply shocks (not covered, lack of time)

Tools, targets and goals

The temporal steps of monetary policy

1. Policy tools
2. Operating targets (instruments)
3. Intermediate targets
4. Final goals



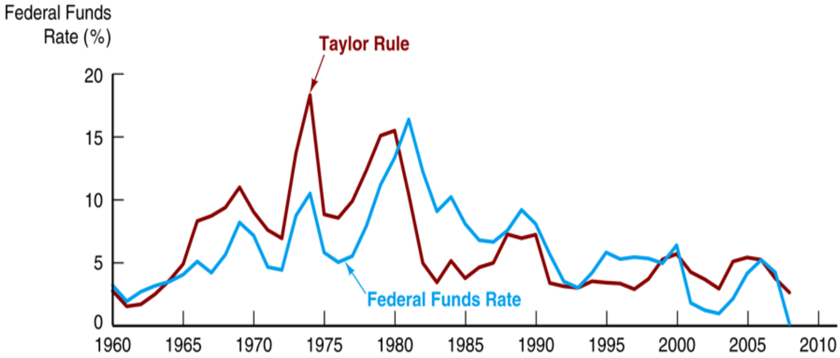
The temporal steps of monetary policy

1. The CB uses (controls) operating targets in order to affect the intermediate targets.
2. Which operating target is better:Money supply? Short term interest rates?
3. Answer: it depends upon the kind of shocks that affect the economy:
    1. Aggregate demand shocks
    2. Money demand shocks
    3. Supply shocks
4. Major result: relative variances of macroeconomic shocks matter for optimal choice of instrument

William Poole (1970). "Optimal choice of monetary policy instruments in a simple stochastic macro model," Staff Studies 57, Federal Reserve Board, Washington.

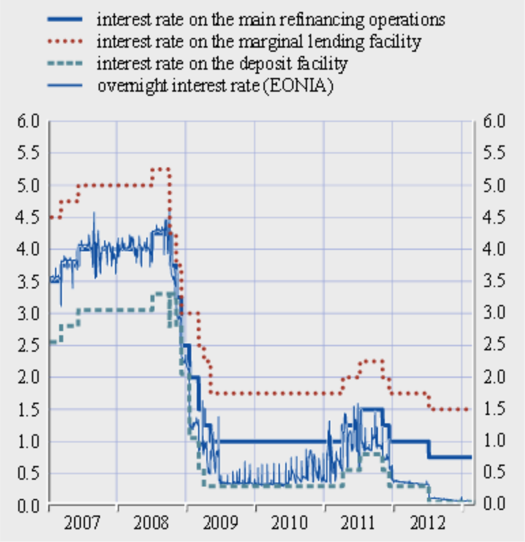
The Taylor rule

A famous example of an operating target rule



The ECB example

Source: ECB.
Note: The last observation relates to 26 February 2010.



Sources: ECB and Thomson Reuters.

A simple model with demand shocks

The basic equations of the model

1. Consider an economy with an IS–LM model with Rational Expectations
2. The IS function tells us that

$$y_t = -b(i_t - E_t \pi_{t+1}) + z_t \tag{1}$$

   Positive shocks on the IS (z_t) boost output at given interest rates.
3. The LM curve tells us that real money demand $(m_t - p_t)$ is dependent upon output and the interest rate

$$\underbrace{m_t - p_t}_{\log \ values} = y_t - a i_t + v_t \tag{2}$$

4. Positive shocks on the LM (v_t) boost money demand at given interest rates and output.

Basic assumptions

1. In the old IS–LM framework, prices and expected inflation are kept as constant. So

$$\begin{aligned} P_t &= 1, \quad \text{then} \quad \ln P_t \equiv p_t = 0 \\ E_t \pi_{t+1} &= 0 \end{aligned}$$

2. The policy maker cannot observe the shocks. It can set either the nominal interest rate (i) or the money supply (m).
3. The problem of the policy maker is to choose between these two instruments to minimize the variance of output

$$\min E[y_t]^2, \quad \text{or} \quad \min E[y_t - y^*]^2$$

4. What is best as instrument? Money supply (m) or (i)?

Solving the model

1. With $E_t \pi_{t+1} = 0$, and $p_t = 0$, The IS and LM eq can be written as

$$\begin{aligned} y_t &= -bi_t + z_t \\ m_t &= y_t - ai_t + v_t \end{aligned}$$

2. If there were no shocks, the solutions would be (no subscripts for simplicity)

$$y^* = -bi^* = \frac{b}{a+b} m^*$$

and

$$i^* = -\frac{1}{a+b} m^*$$

3. The objective of the CB is to minimize

$$\min E[y_t - y^*]^2$$

The CB controls (pegs) the interest rate

1. The objective of the CB is

$$\min E[y_t - y^*]^2$$

2. Therefore, the CB sets $i = i^*$, and output is given by

$$\begin{aligned} y &= -bi^* + z \\ &= y^* + z \end{aligned}$$

3. Then, it is immediate to see that

$$E[y - y^*]^2_{i^*} = \underbrace{Var(z)}_{\sigma_z^2}$$

The CB controls (pegs) the money stock (m)

1. Now the CB instead pegs the money stock $(m = m^*)$.
2. From the IS we know that

$$y = -bi + z$$

3. And from the LM we know that

$$i = \frac{1}{a}(y - m + v)$$

4. Then we have

$$\begin{aligned} y &= -\frac{b}{a}(y - m^* + v) + z \\ & \quad ... \\ &= y^* - \frac{b}{a+b}v + \frac{a}{a+b}z \end{aligned}$$

5. Then, it is immediate to see that

$$E[y - y^*]^2_{m^*} = \left(\frac{b}{a+b}\right)^2 \sigma_v^2 + \left(\frac{a}{a+b}\right)^2 \sigma_z^2$$

Comparing the instruments

The comparison of both instruments is easy

$$\begin{aligned}
E[y-y^*]^2_{i^*} &= E[y-y^*]^2_{m^*} \\
\sigma_z^2 &= \left(\frac{b}{a+b}\right)^2 \sigma_v^2 + \left(\frac{a}{a+b}\right)^2 \sigma_z^2 \\
(a+b)^2\sigma_z^2 &= b^2\sigma_v^2 + a^2\sigma_z^2 \\
& \dots \\
(2ab+b^2)\sigma_z^2 &= b^2\sigma_v^2 \\
\sigma_z^2 &= \underbrace{\left(\frac{b}{2a+b}\right)}_{\phi<1}\sigma_v^2
\end{aligned}$$

Comparing the instruments: summary

$$\begin{aligned} E[y-y^*]^2_{i^*} &= E[y-y^*]^2_{m^*} \\ \sigma^2_z &= \underbrace{\left(\frac{b}{2a+b}\right)}_{\phi<1}\sigma^2_v \end{aligned}$$

1. Hence, choose an interest rate targeting procedure whenever there is
   1. relatively high money demand volatility
   2. relatively low aggregate demand volatility
2. In general targeting the interest rate is preferable if IS shocks are small, money demand is not sensitive to the interest rates (a is small), output is sensitive to the interest rate in IS (b is large):

Extending the model: monetary base as potential instrument

Monetary base as an instrument

1. Central banks control the money base, but not M (for example, $M3$) as in the model
2. Extend the model with endogenous money supply (M):

$$m_t = h_t + ei_t + \omega_t \tag{3}$$

   h_t is the the monetary base.

3. Notice that the money multiplier is given by (expressed in logs)

$$m_t - h_t = ei_t + \omega_t$$

   $\omega_t$is mean-zero money multiplier shock.

4. Money multiplier is increasing in interest rate (banks want to lend more/consumers want to hold less cash)

Solving the model

1. The three fundamental equations are now

$$\begin{aligned} y_t &= -bi_t + z_t && \text{(IS)} \\ m_t &= y_t - ai_t + v_t && \text{(LM)} \\ m_t &= h_t + ei_t + \omega_t && \text{(M Multiplier)} \end{aligned}$$

2. The logic now is

$$\begin{aligned} &y \text{ depends on } i \\ &i \text{ depends on } m \\ &m \text{ depends on } h \end{aligned}$$

3. The objective is to express y, i, m as determined only by h (which is controlled by the CB)

Solving the model (cont.)

1. If there were no shocks, the solutions would be (no subscripts for simplicity, see **Appendix 1** for details)

$$\begin{aligned} y^* &= -bi^* = \frac{b}{a+b+e}h^* \\ i^* &= -\frac{1}{a+b+e}h^* \\ m^* &= h^* + ei^* = \frac{a+b}{a+b+e}h^* \end{aligned} \tag{4}$$

2. The objective of the CB is to minimize

$$\min E[y_t - y^*]^2$$

The CB controls (pegs) the interest rate

1. The same results as above
2. The solution is totally given by the IS function.
3. The CB sets $i = i^*$, and output is given by

$$\begin{aligned} y &= -bi^* + z \\ &= y^* + z \end{aligned}$$

4. Then, it is immediate to see that

$$E[y - y^*]^2_{i^*} = \underbrace{Var(z)}_{\sigma_z^2}$$

The CB controls (pegs) the monetary base (h)

1. Now the CB pegs the monetary base $(h = h^*)$.
2. Then, the Money Multiplier eq. comes as

$$m = h^* + ei + \omega$$

3. But from the LM eq. we know that

$$m = y - ai + v$$

4. Therefore, by equating both previous eq. we get

$$h^* + ei + \omega = y - ai + v$$

5. From where we can obtain

$$i = \frac{1}{a+e}(y - h^* + v - \omega)$$

The CB controls (pegs) the monetary base (h)

1. Now, from the equilibrium in the money market we know that

$$i = \frac{1}{a+e}(y - h^* + v - \omega)$$

2. Then, we should use the IS function to solve the entire problem

$$\begin{aligned} y &= -bi + z \\ &= -\frac{b}{a+e}(y - h^* + v - \omega) + z \end{aligned} \quad (5)$$

3. Solving for y, leads to (see **Appendix 2** for details)

$$y = y^* - \frac{b}{a+b+e}v + \frac{b}{a+b+e}\omega + \frac{a+e}{a+b+e}z \quad (6)$$

4. Then, it is immediate to see that

$$E[y - y^*]^2_{h^*} = \left(\frac{b}{a+b+e}\right)^2 \left(\sigma_v^2 + \sigma_\omega^2\right) + \left(\frac{a+e}{a+b+e}\right)^2 \sigma_z^2$$

Comparing the instruments

1. Now it is easy to see which instrument is better

$$
\begin{aligned}
E[y-y^*]^2_{i^*} &= E[y-y^*]^2_{h^*} \\
\sigma_z^2 &= \left(\frac{b}{a+b+e}\right)^2\left(\sigma_v^2+\sigma_\omega^2\right)+\left(\frac{a+e}{a+b+e}\right)^2\sigma_z^2 \\
(a+b+e)^2\sigma_z^2 &= (b)^2\left(\sigma_v^2+\sigma_\omega^2\right)+(a+e)^2\sigma_z^2 \\
&\ldots \\
\sigma_z^2 &= \underbrace{\left(\frac{b}{2\left(a+e\right)+b}\right)}_{\phi<1}\left(\sigma_v^2+\sigma_\omega^2\right)
\end{aligned}
$$

2. Hence, choose an interest rate targeting procedure whenever there is
   1. relatively high money demand volatility
   2. relatively high money multiplier volatility
   3. relatively low aggregate demand volatility

Extending the model: introducing supply shocks

— No time available for covering this point . Read pages 18-23 in the bibliography if you are curious about this point. But reading is not compulsory—

Appendix 1

1. The 3 eq. with no shocks are

$$\begin{aligned} y &= -bi && \text{(IS)} \\ m &= y_t - ai && \text{(LM)} \\ m &= h + ei && \text{(MM)} \end{aligned}$$

2. Using the LM and the MM eq., we get

$$i = \frac{1}{a+e}(y-h)$$

3. Now insert this result into the IS eq.

$$y = -bi = \frac{b}{a+b+e}h$$

4. Then, it is easy to write back m and i also as functions of h

$$i = -\frac{1}{a+b+e}h \quad , \quad m = h + ei = \frac{a+b}{a+b+e}h.$$

Appendix 2

1. How to get from eq. (5) to eq. (6)? From eq. (5), we have

$$y = -\frac{b}{a+e}(y - h^* + v - \omega) + z$$

2. Solving for y, leads to

$$\begin{aligned} y(1 + \frac{b}{a+e}) &= \frac{b}{a+e}h^* - \frac{b}{a+e}v + \frac{b}{a+e}\omega + z \\ y &= \frac{b}{a+b+e}h^* - \frac{b}{a+b+e}v + \frac{b}{a+b+e}\omega + \frac{a+e}{a+b+e}z \end{aligned}$$

3. Now, using the result in eq. (4)

$$y^* = \frac{b}{a+b+e}h^*$$

4. We arrive at eq. (6)

$$y = y^* - \frac{b}{a+b+e}v + \frac{b}{a+b+e}\omega + \frac{a+e}{a+b+e}z$$