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Chapter 3
GENERAL EQUILIBRIUM IN A TWO-PERIOD ECONOMY

Savings are an important determinant of aggregate investment. In fact, aggregate investment is equal to aggregate savings when the economy is closed. In turn, aggregate investment is an important driving force of business cycles and economic growth. In this chapter we extend the two-period model and analyze some macroeconomic issues.

The extensions endogenize the income levels and the real interest rate, i.e., we determine these variables in the model rather than leave them exogenous. We also introduce the labor supply decision and endogenize the real wage rate. The real interest rate and the real wage rate are the "prices" of productive factors, the real interest rate being the "price" of capital and the real wage rate being the "price" of labor. Such a model with endogenous income levels and endogenous factor prices is called a general equilibrium model.

A general equilibrium model is useful for studying business cycles because it explicitly reveals the relationships between macroeconomic variables. In particular, the real interest rate serves as both the intertemporal goods price and the cost of capital. This dual role of the interest rate links the goods market to the capital market explicitly so that households' decisions in the goods market affect the factor market as well.

To introduce the notion of equilibrium, we proceed gradually by endogenizing only the wage rate first and then endogenizing both the wage rate and the real interest rate.

3.1 A Model with Endogenous Labor Income

To describe an equilibrium in an economy, we follow the following two steps. First, we let individuals take all prices as given and choose their optimal decisions on consumption, savings, etc. Second, we ask that prices be at such levels that there is an equilibrium in every market, i.e., demand equals supply in every market. This description may sound vague at this point, but the important thing to remember is that these two steps must be done in exactly the described order. One should never reverse the order.

3.1.1 Production and Labor Employment

The economy consists of a large number of households. The total number of households is normalized to be 1 (e.g., 1 million or 1 billion). All households are the same and so we can arbitrarily pick one of them and call it the representative household.*

The representative household starts period 0 with an endowed wealth y_0. From this the household chooses the consumption level c_0 and savings s_0. The household's budget constraint in period 0 is the same as before:

$$c_0 + s_0 \leq y_0.$$

In period 1, the household has two additional income sources besides the income from savings. One is labor income. Let l_1^s be the amount of labor supplied by the representative household in period 1, where the superscript s indicates "supply". Let w be the real wage in period 1, which will be determined endogenously later. Labor income is $y_1 = wl_1^s$. The second additional income source is dividend. We assume that each household owns a fully diversified portfolio of shares of firms, which yields a dividend D in period 1. The household's budget constraint in period 1 is

$$c_1 \leq Rs_0 + wl_1^s + D.$$

For this subsection, let us assume that the gross real interest rate is exogenous in the economy, given by a constant $R > 0$. For example, the households in this economy may borrow and lend in the international capital market; If the economy is small relative to the world market, the economy cannot affect the world real interest rate. The household's intertemporal budget constraint is

$$c_0 + \frac{c_1}{R} \leq y_0 + \frac{wl_1^s + D}{R}. \tag{3.1}$$

The household's labor supply decision is an optimal trade-off between labor income and the utility from leisure. For this decision to be non-trivial, the household must value leisure. Normalize the total amount of the household's time to 1. The

*Of course, households are different from each other in reality. The representative household can be regarded as the average household in the economy. If we are interested only in the aggregate behavior of the economy, it simplifies matters by analyzing only the representative household.

amount of leisure in period 1 is $x = 1 - l_1^s$ and the utility that the household obtains from leisure is $u_0 - \Phi(1-x)$, where $u_0 > 0$. For simplicity, we assume that the household's intertemporal utility is

$$u(c_0, c_1, x) = U(c_0) + \beta[U(c_1) + u_0 - \Phi(1-x)],\ \ \beta \in (0,1).$$

Note that the intertemporal utility function is time-additive. Also, the utilities from leisure and consumption in period 1 are separable from each other.

For the intertemporal utility function to have positive and diminishing marginal utility of consumption, we need $U' > 0$ and $U'' < 0$. Similarly, we require that the marginal utility of leisure be positive and diminishing, i.e., $\Phi' > 0$ and $\Phi'' > 0$.[†] To rule out the case in which the optimal consumption level is 0, we assume that the marginal utility of consumption is very large when consumption is 0. Similarly, we impose conditions on $\Phi'(0)$ to guarantee that the optimal choice of labor supply is positive. These assumptions are collected below.

Assumption 1 *The functions U and Φ satisfy the following conditions:*
(i) $U' > 0, U'' < 0$;
(ii) $\Phi' > 0, \Phi'' > 0$;
(iii) $\lim_{c\to 0} U'(c) = \infty$;
(iv) $\Phi'(0) = 0$.

The positive number u_0 is immaterial for the household's decision, and so we omit it. Also, it is convenient to work with the labor supply choice rather than the leisure choice. For these reasons, we use the following intertemporal utility function:

$$u(c_0, c_1, l_1^s) = U(c_0) + \beta[U(c_1) - \Phi(l_1^s)],\ \ \beta \in (0,1). \tag{3.2}$$

The household's decisions, (c_0, c_1, l_1^s), solve the following maximization problem:

$$(PH) \qquad \max_{(c_0, c_1, l_1^s)} U(c_0) + \beta[U(c_1) - \Phi(l_1^s)] \text{ s.t. } (3.1).$$

It is important to emphasize that the household takes the real wage rate as given in this maximization problem, because each household is too small relative to the economy to have any influence on the price of labor.[‡] One output of the

[†]The assumption $\Phi' > 0$ states that people do not like to work if they can obtain income freely. They choose to work because they need the income to sustain consumption.

[‡]Households also take dividends D as given, since dividends are generated by all firms' shares of which the household is one of many shareholders.

household's problem (PH) is the supply function of labor, as we will show later. If we can characterize the demand function for labor, then the real wage rate can be determined by equating demand and supply in the labor market.

Firms demand for labor. Assume that the economy consists of a large number of identical firms in the economy, the number of which is normalized to 1 (again, 1 million or 1 billion). Firms hire labor from many households to produce output in period 1, each hiring l_1^d amount of labor, where the superscript d indicates "demand". Since capital is not the focus of our discussion at this moment, let us assume that firms do not need capital to produce. If savings are positive, the households lend them to the rest of the world; if savings are negative, the households borrow from the rest of the world.

The amount of output is given by a production function $F(l_1^d)$, which is strictly increasing and concave. Also, $F(0) = 0$ and $F'(0) = \infty$. The first condition requires that labor be necessary for producing output and the second condition requires that the optimal labor input be positive.

After production, the firm pays the workers the wage and the shareholders (the households) dividends. The amount of dividends is equal to the firm's net profit. Since all firms are identical in this economy, their dividends to the shareholders are the same and hence are equal to D. Then,

$$D \equiv F(l_1^d) - wl_1^d.$$

The firm chooses l_1^d to maximize net profit D, i.e.,

$$(PF) \quad \max_{l_1^d} \ D.$$

Each firm takes the wage rate as given. This is because there are a large number of firms in the economy, each of which has negligible influence on the wage rate. The solution to (PF) yields the demand function for labor.

3.1.2 Optimal Decisions of Households and Firms

To solve for the household's optimal decisions, let λ be the Lagrangian multiplier of the intertemporal budget constraint, (3.1). The Lagrangian of the household's maximization problem, (PH), is

$$\mathcal{L} \equiv U(c_0) + \beta[U(c_1) - \Phi(l_1^s)] + \lambda\left(y_0 + \frac{wl_1^s + D}{R} - c_0 - \frac{c_1}{R}\right).$$

The first-order conditions for c_0, c_1 and l_1^s are, respectively,

$$U'(c_0) - \lambda = 0, \tag{3.3}$$

$$\beta U'(c_1) - \frac{\lambda}{R} = 0, \tag{3.4}$$

$$-\beta \Phi'(l_1^s) + \frac{\lambda w}{R} = 0. \tag{3.5}$$

Note that (3.3) holds with equality rather than "$\leq$" because the condition (iii) in Assumption 1 guarantees that (3.3) can be satisfied by a positive level of c_0. (Recall that the first order condition of c_0 has the form "$\leq$" only when the optimal level of c_0 is not zero.) Similarly, under Assumption 1, (3.4) is satisfied by a positive level of c_1 and (3.5) is satisfied by a positive quantity of labor supply.

The household's decisions generate the demand functions for goods and the supply functions of productive factors. For the demand functions for goods, combine (3.3) and (3.4). We have the familiar condition for optimal consumption:

$$\frac{U'(c_0)}{\beta U'(c_1)} = R. \tag{3.6}$$

With this condition and the household's intertemporal budget constraint, we can solve the optimal levels of consumption, (c_0^*, c_1^*), as functions of R (and w). These are demand functions for goods because R is the relative price of goods between the two periods. The household's decision also implies an optimal level of savings, $s_0^* = y_0 - c_0^*$.

The condition (3.5) characterizes the household's labor supply decision. It requires that the marginal cost of supplying labor, $\beta \Phi'$, be equal to the marginal benefit of labor supply. With an additional increment in labor supply, the household obtains the wage w. Such a benefit is λw in terms of period-1 utility and hence $\lambda w / R$ in terms of period-0 utility.

Because income is used to increase consumption, we can also connect the household's labor supply decision to consumption by, using (3.4) to eliminate λ from (3.5):

$$\frac{\Phi'(l_1^s)}{U'(c_1)} = w. \tag{3.7}$$

The left-hand side of this equation is the marginal rate of substitution between leisure and consumption. If the household works for one additional unit of time, the added income must generate at least $\Phi'(l)/U'(c)$ units of consumption, in order for the household to feel not worse off than before. The right-hand side of the equation is how much an additional unit of time can actually earn in the (labor) market. Thus, informally, the labor supply decision equates how much the household would like to get from an additional unit of labor supply to what is available in the market.

The equation (3.7) is an implicit labor supply function. To see this, let $\phi(\cdot)$ be the inverse function of Φ', i.e., $\phi(\Phi'(l)) = l$ for all l. Eq. (3.5) can be rewritten as

$$l_1^s = \phi\left(wU'(c_1)\right). \tag{3.8}$$

Since Φ' is an increasing function, so is ϕ. Eq. (3.8) states that labor supply is an increasing function of the real wage rate, given the consumption level.

The demand function for labor comes from the firm's maximization problem, (PF). The first-order condition of the firm's problem is

$$F_l(l^d) - w = 0. \tag{3.9}$$

That is, the optimal demand for labor equates the marginal product of labor to the marginal cost of labor. This condition implicitly gives l^d as a function of w – the demand function for labor. Since the marginal product of labor diminishes in the labor input, the above condition determines the demand for labor as a decreasing function of the real wage rate.

3.1.3 Competitive Equilibrium

A competitive equilibrium in this economy consists of the household's decisions $(c_0^*, c_1^*, s_0^*, l_1^{s*})$, the firm's decision l_1^{d*}, the real wage rate w and dividends D such that the following conditions hold: (i) Given (w, D), the household's decisions solve the household's intertemporal maximization problem, (PH); (ii) Given w, the firm's labor demand solves the firm's profit maximization problem, (PF), and the amount of dividends is equal to the firm's net profit; (iii) All markets clear (i.e., prices equate demand and supply in the markets):
(iiia) $c_0^* + s_0^* = y_0$;
(iiib) $c_1^* = F(l_1^{d*}) + Rs_0^*$;
(iiic) $l_1^{d*} = l_1^{s*}$.

The equilibrium is competitive because individuals take prices as given and prices clear the markets. There are four markets in the economy. One is the capital market in period 1, for which we did not specify the clearing condition because the economy lends all capital to the rest of the world. The other three markets are the goods market in period 0, the goods market in period 1 and the labor market in period 1. The equilibrium conditions for these three markets are given by (iiia) – (iiic), respectively.

In the goods market in period 0, the demand for goods is the sum of households' demand for goods and the demand for the economy's capital by the rest of the world, the latter of which is equal to the economy's savings. The supply of goods is exogenously given as y_0. So, the goods market clears in period 0 if $c_0^* + s_0^* = y_0$. In the goods market in period 1, the demand for goods is the households' demand for consumption. The supply of goods is output from production and payment to the household's capital by the rest of the world. So, the goods market clears in period 1 if $c_1^* = F(l_1^{d*}) + Rs_0^*$. The supply of labor and the demand for labor in period 1 are l_1^s and l_1^d, respectively, and so the labor market clears only when the two are equal to each other.

Among the three market clearing conditions, (iiia) simply repeats the household's budget constraint in period 0. Similarly, (iiib) repeats the household's budget constraint in period 1 once the amount of dividend is set to equal to the representative firm's net profit. Only the labor market clearing condition is not redundant.

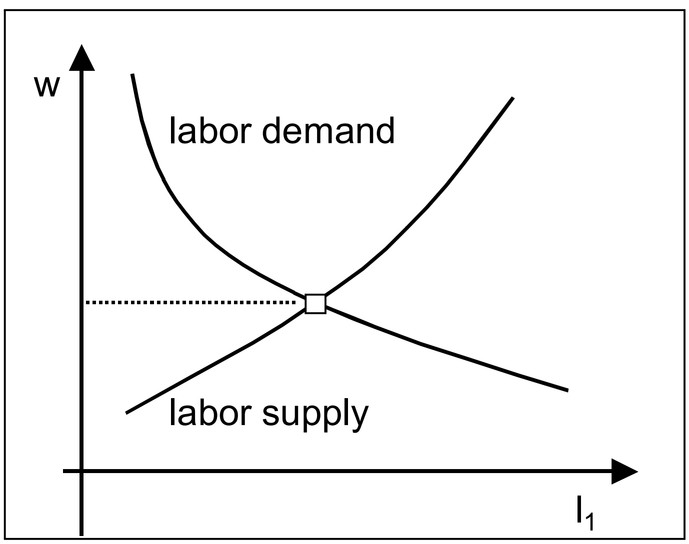

Figure 3.1 The labor market

We can depict the labor market clearing condition in Figure 3.1 for any given (positive) λ. The upward sloping curve is the labor supply curve (3.8); the downward sloping curve is the labor demand curve given implicitly by (3.9). The labor market clears at the intersection between these curves, which gives the equilibrium wage and equilibrium labor employment.

3.1.4 Equilibrium Solution

The above definition of an equilibrium suggests the following way to solve for an equilibrium. First, taking w as given, we solve for the demand and supply functions of goods and factors. Second, we equate the demand and supply in each market to solve for the prices. Then we substitute such prices back into the demand or supply functions to obtain equilibrium quantities of consumption, capital and labor employment. Unfortunately, it is difficult to use this approach, because it is not easy to solve explicitly for the demand and supply functions. An alternative way is to derive the first-order conditions from the household's and the firm's problems, taking prices as given, and then substitute the market clearing conditions into those first-order conditions.

We illustrate the alternative approach here. The household's optimal conditions are (3.1), (3.6), and (3.5); the firm's optimal condition is (3.9). Substituting the market clearing conditions (iiia) – (iiic) into these conditions, we have $w = F_l$, $c_0 = y_0 - s_0$, and

$$\frac{U'(y_0 - s_0)}{\beta U'(Rs_0 + F(l_1))} = R, \tag{3.10}$$

$$\Phi'(l_1) = U'(Rs_0 + F(l_1))F_l(l_1). \tag{3.11}$$

We have suppressed the superscripts s and d here because demand equals supply in equilibrium.

Eq. (3.10) states that the marginal rate of substitution between current and future consumption, $U'(c_0)/[\beta U'(c_1)]$, must be equal to the marginal rate of transformation, which is similar to the optimal intertemporal trade-off we explained before. Eq. (3.11) comes from (3.7), with F_l replacing w and $(Rs_0 + F(l_1))$ replacing c_1. The two equations jointly solve for (s_0, l_1). Once this is done, other variables can be recovered accordingly.[§]

Example 1 *Assume that $U(c) = \ln c$, $\Phi(l) = l^2/2$, and $F(l) = Al^{1-\alpha}$, $\alpha \in (0,1)$. Also, assume $y_0 = 0$. Then (3.10) and (3.11) yield:*

$$l_1 = [(1-\alpha)(1+\beta)/\beta]^{1/2}.$$

[§]The solutions to (3.10) and (3.11) are unique. To prove this, rewrite the two equations as $h1(s_0, l_1) = 0$ and $h2(s_0, l_1) = 0$, where $h1(s_0, l_1) = U'(y_0 - s_0) - \beta RU'(Rs_0 + F(l_1))$ and $h2(s_0, l_1) = \Phi'(l_1) - \frac{1}{\beta R}U'(y_0 - s_0)F'(l_1)$. We can verify that $\partial h1/\partial s_0 > 0$ and $\partial h1/\partial l_1 > 0$, implying $ds_0/dl_1 < 0$ along $h1(s_0, l_1) = 0$. That is, $h1(s_0, l_1) = 0$ solves for a negative relationship between s_0 and l_1. Similarly, $\partial h2/\partial s_0 < 0$ and $\partial h2/\partial l_1 > 0$, and so $h2(s_0, l_1) = 0$ solves for a positively relationship between s_0 and l_1. The intersection between these two relationships must then be unique.

Equilibrium wage rate is $w = A\left[\left(\frac{\beta}{1+\beta}\right)^{\alpha}(1-\alpha)^{2-\alpha}\right]^{1/2}$.

Exercise 3.1.1 *Assume that* $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$ $(\sigma > 0)$, $\Phi(l) = l^2/2$, *and* $F(l) = Al^{1-\alpha}$, $\alpha \in (0,1)$. *Also, assume* $y_0 = 0$. *Then, labor employment in equilibrium is*

$$l = \left[\frac{1-\alpha}{\beta R}\left(R + (\beta R)^{1/\sigma}\right)^{\sigma} A^{1-\sigma}\right]^{\frac{1}{1+\alpha+\sigma(1-\alpha)}}.$$

The wage rate is $w = (1-\alpha)Al^{-\alpha}$. *We have,* $dl/dR > 0$ *and* $dw/dR < 0$ *if and only if* $\sigma > 1$.

Before completing this section, we re-examine the labor market equilibrium. In Figure 3.1, which depicts the labor market equilibrium, the supply curve was drawn taking c_1 as given. In equilibrium, however, consumption is determined endogenously. If there is a disturbance in the economy that increases c_1, it will reduce the marginal utility of consumption and increase the cost of leisure relative to consumption. For any given wage rate, this will induce the household to reduce labor supply. That is, the labor supply curve in Figure 3.1 will shift up.

The shift in the labor supply curve can help us understand the effects of an increase in the marginal productivity of labor. Let $F(l) = Af(l)$, where A is the marginal productivity of labor. An increase in A affects both the supply curve and the demand curve. The effect on the labor supply curve comes from the response in consumption. When A increases, output and hence income increases. The resulted increase in consumption c_1 shifts up the labor supply curve. This is an income effect of A on labor supply.[¶] The other effect of an increase in A is that it shifts up the labor demand curve, as the workers are more productive now. This results in a higher wage rate. To equate the marginal rate of substitution between leisure and consumption to this higher wage rate, the household must increases labor supply, i.e., it substitutes away from leisure and into consumption. This is a substitution effect of A on labor supply. Equilibrium labor supply increases if and only if the substitution effect dominates the income effect.

The balance between the two effects depends on the curvature of the utility function, because the income effect does. The more concave the utility function, the larger the decrease in the marginal utility of consumption in response to an increase in consumption, and the stronger the (negative) income effect of A on l. Thus, for

[¶]An equivalent interpretation of the income effect is that the increase in income increases the "consumption" of leisure, since leisure is a normal good.

equilibrium employment to respond positively to an increase in A, the utility function cannot be too concave. In Example 1, the utility function is logarithmic and the two effects exactly cancel out, leaving equilibrium employment constant. Exercise 3.1.1 shows that the substitution effect dominates, and hence equilibrium employment increases in A, if and only if σ is less than 1. A small σ means that the utility function is not very concave.‖

3.2 A General Equilibrium Model

We now endogenize the real interest rate as well as the real wage rate. For the real interest rate to be determined endogenously, the economy as a whole must have non-negligible influence on the real interest rate. (As before, each individual has no control over the real interest rate). To simplify the illustration we assume that the economy is a closed economy, and so the real interest rate is determined completely within the economy.

3.2.1 Households' and Firms' Decisions

The household's decisions are similar to the ones in the last section. The only new element is that the households' decisions on savings determine the supply of capital in the economy. We assume that capital has the same physical form as consumption goods and so they have the same relative price. Let k_1^s be the supply of capital by each household. Then, $k_1^s = s_0$. The household faces the intertemporal budget constraint (3.1) and the maximization problem is (PH), the same as in the last section. The first-order conditions for c_0, c_1 and l_1^s are given by (3.3), (3.4) and (3.5), respectively.

The household's decisions generate the demand functions for goods. To obtain these demand functions, combine (3.3), (3.4) and the household's intertemporal budget constraint to solve the optimal levels of consumption, (c_0^*, c_1^*), as functions of R (and w). These are demand functions for goods because R is the relative price of goods in the two periods.

The household's decisions also generate the supply functions of productive factors. The supply function of labor is implicitly given by (3.5), as explained in the last section. To obtain the supply function of capital, recall that $k_1^s = s_0^*$. Since c_0^* is a function of R and w from the household's optimal consumption choices, k_1^s is a function of R and w. This is a supply function of capital because R is the rental cost

‖Recall that $1/\sigma$ is also the elasticity of intertemporal substitution. Unfortunately, with the functional form $U(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, we cannot distinguish the curvature of the utility function from the elasticity of intertemporal substitution.

of capital.

To find the demand functions for productive factors, consider a representative firm's maximization problem. Firms now employ both capital and labor to produce. In contrast to the last section, the number of firms is not fixed. Rather, firms can freely enter the industry. There is no fixed cost to set up a firm and so the industry is perfectly competitive. Because of perfect competition, the number of firms is not important for the equilibrium (the industry's total demand for factors is). Normalize the number of firms in the economy to 1 (again, 1 million or 1 billion). Each firm rents k_1^d units of capital and hires l_1^d units of labor to produce in period 1. The amount of output is given by a production function $F(k_1^d, l_1^d)$.

After production, the firm pays the workers the wage, the suppliers of capital the rental cost of capital and the shareholders the dividends. To simplify illustration, we assume that capital depreciates completely after production, and so the firm must pay the principal and interest on capital entirely from output. In this case the firm's net profit or dividend is[**]

$$D \equiv F(k_1^d, l_1^d) - wl_1^d - Rk_1^d.$$

The firm chooses (k_1^d, l_1^d) to maximize net profit D, i.e.,

$$\max_{(k_1^d, l_1^d)} D.$$

Since the industry is perfectly competitive, the firm takes factor prices (w, R) as given. The solutions to this problem yields the demand functions for capital and labor.

Before solving for the equilibrium we specify properties of the production function $F(k, l)$, as follows.

(F1) $F(k, l)$ is increasing in k and l, i.e., $F_k > 0$ and $F_l > 0$, where F_k and F_l are the derivatives with respect to k and l respectively: Increasing each input while fixing the other input increases output.[††]

(F2) $F(k, l)$ is concave in each argument, i.e., $F_{kk} > 0$ and $F_{ll} > 0$, where F_{kk} and F_{ll} are the second-order derivatives with respect to k and l respectively: Increasing

[**]If capital depreciates by a δ fraction after production, the firm pays capital from output and the remainder of the capital stock, $(1-\delta)k$. In this case the firm's net profit is $F + (1-\delta)k - wl - Rk$.

[††]We deviate from the earlier notation of derivatives by using the letters of the variables as subscripts rather than 1, 2, etc.

one input successively while fixing the other input increases output by a smaller amount each time.

(F3) $F(k,l)$ exhibits constant returns to scale, i.e., $F(ak,al) = aF(k,l)$ for any $a > 0$: Increasing both inputs by a times increases output by a times.

The assumptions (F1) – (F3) imply that the two productive factors are complementary. That is, if the amount of one input increases, marginal productivity of the other factor increases. Mathematically, complementarity means

(F4) $F_{kl} > 0$.

When a production exhibits constant returns to scale, it is linearly homogeneous. Linear homogeneity implies the following three properties:

(F5) $kF_k + lF_l = F$. This property states that output is exhausted by the marginal products of the two factors. That is, if both factors are paid at their marginal products, the total factor payment is equal to output. This property is necessary for guaranteeing that each firm earns zero net profit (as shown later).

(F6) $F_{kk}F_{ll} - F_{kl}^2 = 0$. This property implies that the production function is not concave in the two arguments jointly, although it is concave in each argument.

(F7) $F(k,l) = lf\left(\frac{k}{l}\right)$ for some function f. Under Assumptions (F1) – (F3), the function f is increasing and concave, i.e., $f' > 0$ and $f'' < 0$.

We do not prove (F4) – (F7) here. Instead, we provide the following two exercises to verify these properties.

Exercise 3.2.1 *Let the production function be the Cobb-Douglas form:*

$$F(k,l) = Ak^{\alpha}l^{1-\alpha}, \quad \alpha \in (0,1), \quad A > 0.$$

Establish the following results.
(i) The function satisfies (F1) – (F3).
(ii) The function satisfies (F4) – (F6).
(iii) The function satisfies (F7), with $f = A(k/l)^{\alpha}$.

Exercise 3.2.2 *Let the production function be* $F(k,l) = A[\alpha k^{\rho} + (1-\rho)l^{\rho}]^{1/\rho}$*, where* $\alpha \in (0,1)$ *and* $\rho < 1$*. This function is the CES function, i.e., the elasticity of substitution between* k *and* l *is constant. Show that the CES function satisfies (F1) – (F7), with* $f = A[\alpha + (1-\alpha)(k/l)^{\rho}]^{1/\rho}$*.*

The demand functions for productive factors come from the firm's maximization problem. The first-order conditions of the firm's problem are

$$F_k - R = 0, \tag{3.12}$$

$$F_l - w = 0. \tag{3.13}$$

These conditions state that the marginal product of each factor should be equal to the corresponding factor price. These conditions implicitly solve k_1^d and l_1^d as functions of (w, R) – the demand functions for factors. For example, since the marginal product of capital is a decreasing function of capital, (3.12) requires that the capital input be a decreasing function of the rental rate of capital, given the labor input.

It is worthwhile noting that each firm's net profit is zero when capital and labor inputs are chosen optimally. This is an implication of free entry into the industry and the linearly homogeneous production function. To see this, note that the total cost of factors is $Rk_1^d + wl_1^d$, which is equal to $F_k k_1^d + F_l l_1^d$ under (3.12) and (3.13). Linear homogeneity implies that $F_k k_1^d + F_l l_1^d = F$ and so the firm's net profit is zero.

3.2.2 Competitive General Equilibrium

A competitive general equilibrium consists of the household's decisions $(c_0^*, c_1^*, s_0^*, l_1^{s*})$, the firm's decisions (k_1^{d*}, l_1^{d*}), and prices (w, R) such that the following conditions hold: (i) Given (w, R), the household's decisions solve the household's intertemporal maximization problem, (PH), and the firm's decisions solve the firm's profit maximization problem, (PF); (ii) The prices (w, R) clear the markets:
(iia) $c_0^* + k_1^{d*} = y_0$;
(iib) $c_1^* = F(k_1^{d*}, l_1^{d*})$;
(iic) $k_1^{d*} = k_1^{s*}$, where $k_1^{s*} = s_0^*$;
(iid) $l_1^{d*} = l_1^{s*}$.

As in the last section, the equilibrium is competitive because individuals take prices as given and prices clear all the markets. There are four markets in the economy, whose equilibrium conditions are given by (iia) – (iid) respectively. In the goods market in period 0, the demand for goods is the sum of households' demand for goods and firms' demand for capital. The supply of goods is exogenously given as y_0.

In the goods market in period 1, the demand for goods is the households' demand for consumption. The supply of goods is output from production. At the beginning of period 1, the markets for productive factors must also be cleared. That is, the demand for capital must be equal to the supply of capital (savings) and the demand for labor must be equal to the household's labor supply.

Among the four market clearing conditions, (iia) and (iib) are redundant. Since $k_1^{d*} = k_1^{s*} = s_0^*$, (iia) simply repeats the household's budget constraint in period 0. Similarly, (iib) repeats the household's budget constraint in period 1. This is a general principle called the Walras Law: In each period, if all the budget constraints hold with equality in a period, then one of the market clearing conditions in that period is redundant.

To solve for the general equilibrium, we derive the first-order conditions from the household's and the firm's problems, taking (w, R) as given, and then substitute the market clearing conditions into those first-order conditions. The household's optimal conditions are (3.1), (3.6), and (3.5); the firm's optimal conditions are (3.12) and (3.13). Substituting the market clearing conditions (iia) – (iid) into these conditions, we have $R = F_k$, $w = F_l$, $c_0 = y_0 - k_1$, and

$$\frac{U'(y_0 - k_1)}{\beta U'(F(k_1, l_1))} = F_k(k_1, l_1), \tag{3.14}$$

$$\Phi'(l_1) = U'(F(k_1, l_1))F_l(k_1, l_1). \tag{3.15}$$

In these equations, we have suppressed the superscripts s and d because demand equals supply in equilibrium.

These two conditions differ from their counterparts, (3.10) and (3.11), in two aspects. First, the marginal rate of transformation is now the marginal product of capital, because each unit of goods saved in period 0 can be transformed into productive capital which produces F_k units of goods in period 1. Second, consumption in period 1 is now F rather than $F + Rs_0$. This is because we now assume that the economy is closed to the rest of the world.

Eqs. (3.14) and (3.15) determine the capital and labor inputs in equilibrium, as illustrated in the following example.

Example 2 *Assume that* $U(c) = \ln c$, $\Phi(l) = l^2/2$, *and* $F(k, l) = Ak^{\alpha}l^{1-\alpha}$, $\alpha \in (0, 1)$. *Then (3.14) and (3.15) yield:*

$$k_1 = \frac{\alpha\beta}{1 + \alpha\beta} y_0, \quad l_1 = (1 - \alpha)^{1/2}.$$

Consumption and factor prices are as follows:

$$c_0 = \frac{y_0}{1+\alpha\beta}, \quad c_1 = A\left(\frac{\alpha\beta y_0}{1+\alpha\beta}\right)^{\alpha}(1-\alpha)^{(1-\alpha)/2},$$

$$w = (1-\alpha)^{1-\frac{\alpha}{2}} A\left(\frac{\alpha\beta y_0}{1+\alpha\beta}\right)^{\alpha},$$

$$R = \alpha A\left(\frac{\alpha\beta y_0}{1+\alpha\beta}\right)^{\alpha-1}(1-\alpha)^{(1-\alpha)/2}.$$

With the general equilibrium model we can examine the effects of shocks to the economy and check whether a shock can generate the co-movements among aggregate variables that are characteristics business cycles.

3.3 Productivity Shocks

Productivity shocks are often very persistent. To emphasize the persistence, we examine a permanent productivity shock here. We re-interpret the income level y_0. Rather than treating it as any income, we now suppose that it is produced by a fixed initial capital stock, k_0, and a fixed labor input, 1, in period 0. That is, $y_0 = Ak_0^{\alpha}1^{1-\alpha} = Ak_0^{\alpha}$. The choices in period 1 are the same as in the last section. A positive, permanent productivity shock can be captured by an increase in total factor productivity, A. More precisely, suppose that the household realizes an increase in A at the beginning of period 0 and expects it to last through the lifetime. The shock is not anticipated before period 0.

To illustrate the macroeconomic effects of this positive productivity shocks, consider Example 2. The equilibrium values of macroeconomic variables are obtained by substituting $y_0 = Ak_0^{\alpha}$ into the solutions in that example. We describe the response of each variable to the shock.

The responses in period 0 are as follows. Consumption increases with the productivity shock, because the positive shock increases the household's income. Similarly, savings increase with the shock. Since investment (k_1) is equal to savings, investment also increases. Thus, aggregate consumption, investment, and output respond to the shock in the same direction in period 0 – a positive co-movement. In contrast, labor supply does not change with the shock, because we have exogenously fixed the labor supple level at 1 in period 0.

In period 1, investment is 0 because period 1 is the last period of the households life. Consumption increases with the productivity shock because output/income increases in period 1. Again, aggregate consumption and output exhibit positive co-movement. There are two reasons for output to increase in period 1. One is the direct effect of the productivity shock. Since the shock lasts through period 1, output would increase in period 1 even if capital and labor inputs were fixed. The second reason for output to increase is that, by increasing savings and hence investment in period 0, the productivity shock increases the capital input in period 1. In other words, the productivity shock in period 0 is propagated into period 1 by the positive response of investment in period 0.

The co-movement between consumption and investment is consistent with business cycle facts. Thus, persistent productivity shocks are a potent explanation for business cycles. There are, however, a few exceptions. First, the real wage also responds strongly and positively to the productivity shock in period 1, producing a strong and positive co-movement between the real wage and output. This pro-cyclical movement of the real wage arises because the real wage rate is equal to the marginal product of labor which increases in response to the positive productivity shock. In Figure 3.1, a positive productivity shock shifts the labor supply curve up, thus producing higher equilibrium wage. The strongly pro-cyclical real wage is not a stylized fact of the data. On the contrary, the US data shows that the contemporaneous correlation between the real wage and output is almost zero, suggesting that shocks other than the productivity shocks might be important for explaining the behavior of the real wage.

Second, the labor supply is constant in period 1. This is a special feature of Example 2, not a general feature. As discussed immediately after Exercise 3.1.1, an increase in the marginal productivity of labor have two opposing effects on equilibrium employment. One is to increase labor supply by increasing income (the income effect) and the other is to induce the household to substitute away from leisure and into consumption by increasing the return to employment (the substitution effect). With the logarithmic utility function in Example 2, these two effects exactly cancel out.

With more general functional forms, equilibrium employment may increase or decrease in A, depending on the curvature of the utility function. The following exercises illustrate this result.

Exercise 3.3.1 *Assume that* $U(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ $(\sigma > 0)$, $\Phi(l) = l^2/2$, *and* $F(k,l) = Ak^{\alpha}l^{1-\alpha}$, $\alpha \in (0,1)$. *The elasticity of intertemporal substitution is* $1/\sigma$ *and the logarithmic utility function is a special case with* $\sigma = 1$. *From (3.14) and (3.15) show* *that labor supply in period* 1 *satisfies* $LHS(l,A) = RHS(l)$, *where*

$$LHS(l,A) \equiv k_0^{\alpha} A^{1+\frac{1}{\alpha\sigma}} (1-\alpha)^{\frac{-1}{\alpha(\sigma-1)}} l^{\frac{1+\sigma}{\alpha(\sigma-1)}-1} - A^{-\frac{\sigma-1}{\alpha\sigma}},$$

$$RHS(l) \equiv (\alpha\beta)^{-\frac{1}{\sigma}} (1-\alpha)^{-\frac{1-\alpha}{\alpha\sigma}} l^{\frac{(1-\alpha)(1+\sigma)}{\alpha\sigma}}.$$

Establish the following results:

(i) $LHS(l,A)$ *decreases in* l *if and only if* $\sigma < 1$.

(ii) $LHS(l,A)$ *increases in* A *for given* l, *provided* $c_0 > 0$.

(iii) $RHS(l)$ *increases in* l.

(iv) If $\sigma < 1$, *then equilibrium employment increases in* A *if* $\sigma < 1$.

(v) For σ *greater than* 1 *but sufficiently close to* 1, *equilibrium employment decreases with* A.

Exercise 3.3.2 *Let the intertemporal utility function be* $u(c_0, c_1, l_1) = c_0^{\theta} c_1^{1-\theta} - l_1^2/2$ $(\theta \in (0,1))$ *and the production function be* $F(k,l) = Ak^{\alpha} l^{1-\alpha}$ $(\alpha \in (0,1))$. *Show that equilibrium employment is*

$$l_1 = \left[(1-\theta)(1-\alpha)c_0^{\theta} A^{1-\theta} k^{\alpha(1-\theta)}\right]^{1/[1+\alpha+\theta(1-\alpha)]},$$

where $k = \alpha(1-\theta)y_0/[\theta + \alpha(1-\theta)]$ *and* $c_0 = y_0 - k$. *Clearly,* $dl_1/dA > 0$ *if and only if* $\theta < 1$.

Let us return to Example 2 and examine how other variables respond to the productivity shock. The real interest rate responds positively to the productivity shock. Although increased productivity in period 0 increases the capital stock for period 1 and tends to reduce the real interest rate through diminishing marginal product of capital, the higher productivity in period 1 directly increases the marginal product of capital in period 1. The latter effect dominates, at least in Example 2.

To link the exercise in this section to the analysis on savings in the last chapter, we note that the productivity shock changes both the real interest rate and the income profile, which have opposite effects on savings in period 0. The productivity shock increases period 1 income by more than period 0 income, because it increases both the labor income and the capital income in period 1. This rising income profile induces the household to save less, in an attempt to smooth consume intertemporally. The increase in the real interest rate, however, induces the household to save more. If the productivity shocks did not affect period 0 income, then these two effects on savings would exactly balance each other in Example 2, leaving savings unchanged. Put differently, savings increase in Example 2 in response to the productivity shock primarily because period 0 income has increased.

3.4 Government Spending Shocks

Although persistent productivity shocks are a potent explanation for business cycles, not all of their predictions are consistent with data. In particular, the strong, positive response of the real wage rate to productivity shocks is counter-factual. This motivates the examination of other types of shocks, such as shocks to government spending.

To see how government spending shocks might be useful in explaining the behavior of the real wage, let us reconsider Figure 3.1. A government spending shock affects the equilibrium wage primarily by shifting the labor supply curve. An increase in government spending in period 0, for example, reduces private consumption, as we will show later. This increases the marginal utility of income, because the latter is equal to the marginal utility of consumption. The labor supply curve shifts down, and so the real wage falls in equilibrium. In the event of a positive productivity shock, a concurrent government spending shock can mitigate the positive effect of the productivity shock on the real wage.

To isolate the effects of a government spending shock, we now assume that there is no productivity shock. We also adopt the extreme assumption that government spending does not generate any benefit to the households, and so the households' intertemporal utility is still represented by (3.2).[‡‡] Let g_i be the level of government spending per household in period i $(= 0, 1)$. The government finances the spending g_i by collecting lump-sum taxes τ_i per household in each period i $(= 0, 1)$. The government budget is balanced in each period, i.e.,

$$g_i = \tau_i.$$

In this model, government debt does not affect equilibrium allocations, because the Ricardian equivalence holds (see Exercise 3.4.1 later).

With lump-sum taxes, the household's budget constraints in the two periods are, respectively,

$$c_0 + s_0 \leq y_0 - \tau_0,$$

$$c_1 \leq Rs_0 + wl_1^s - \tau_1.$$

[‡‡]In reality government spending does generate benefit to households but the marginal benefit is less than that of private consumption. With this more realistic assumption the analytical results of the model should still be valid.

The intertemporal budget constraint of the household is

$$c_0 + \frac{c_1}{R} \leq \left(y_0 + \frac{w l_1^s}{R} \right) - \left(\tau_0 + \frac{\tau_1}{R} \right).$$

This budget constraint states that the present value of the household's consumption cannot exceed the household's present value of income minus the present value of tax liabilities. Because income minus taxes is the household's "disposable income", the present value of consumption cannot exceed the present value of disposable income.

Because the taxes are lump-sum, the household's optimal decisions still obey the first-order conditions (3.3) – (3.5). (Of course, the equilibrium quantities of the variables in those conditions are different.) Similarly, the firm's optimal decisions are characterized by (3.12) and (3.13).

The equilibrium definition is slightly different now, because the economy has both the private sector and the government sector. The differences appear in the market clearing conditions for the goods market. Total demand for goods is $c_0^* + k_1^{d*} + g_0$ in period 0 and $c_1^* + g_1$ in period 1. Thus, the conditions (iia) and (iib) in the above definition of a general equilibrium are now replaced by $c_0^* + k_1^{d*} + g_0 = y_0$ and $c_1^* + g_1 = F(k_1^{d*}, l_1^{d*})$, respectively. In equilibrium we have $R = F_k$, $w = F_l$, $c_0 = y_0 - k_1 - g_0$, and

$$\frac{U'(y_0 - k_1 - g_0)}{\beta U'(F(k_1, l_1) - g_1)} = F_k(k_1, l_1), \tag{3.16}$$

$$\Phi'(l_1) = U'(F(k_1, l_1) - g_1) F_l(k_1, l_1). \tag{3.17}$$

Eqs. (3.16) and (3.17) are similar to (3.6) and (3.15).

The procedures for solving for the equilibrium is as follows. First, we use (3.16) and (3.17) to determine k_1 and l_1 jointly. Second, factor prices are $R = F_k$ and $w = F_l$. Third, consumption is $c_0 = y_0 - g_0 - k_1$ in period 0 and $c_1 = F(k_1, l_1) - g_1$ in period 1; savings are $s_0 = k_1$.

Exercise 3.4.1 *This exercise demonstrates the Ricardian equivalence which states that, for any given government spending profile, the government's choice between tax financing and debt financing is irrelevant for the equilibrium. To show the Ricardian equivalence, suppose that the government issues debt in period* 0 *and repays in period* 1*. The government's budget constraint is* $g_0 = \tau_0 + B$ *in period* 0 *and* $g_1 = \tau_1 - RB$ *in period* 1*, where* B *is the amount of debt issued in period* 0*. Reformulate the household's and the firm's maximization problems and show that the equilibrium is unaffected by the debt. Explain intuitively why the Ricardian equivalence holds.*

Example 3 *Assume* $U(c) = \ln c$, $\Phi(l) = l^2/2$ *and* $F(k, l) = Ak^{\alpha}l^{1-\alpha}$ $(\alpha \in (0, 1))$. *Consider the case where government spending is proportional to aggregate output per household, i.e.,* $g_0 = \gamma_0 y_0$ *and* $g_1 = \gamma_1 F(k_1, l_1)$ *with* $\gamma_0, \gamma_1 \in (0, 1)$. *Show that factor inputs in equilibrium are*

$$k_1 = \frac{\alpha\beta(1-\gamma_0)}{1-\gamma_1+\alpha\beta} y_0, \quad l_1 = \left(\frac{1-\alpha}{1-\gamma_1}\right)^{1/2}.$$

Other variables have the following values in equilibrium:

$$c_0 = \frac{(1-\gamma_0)(1-\gamma_1)y_0}{1-\gamma_1+\alpha\beta},$$

$$c_1 = (1-\gamma_1)^{(1+\alpha)/2} A \left(\frac{\alpha\beta(1-\gamma_0)y_0}{1-\gamma_1+\alpha\beta}\right)^{\alpha} (1-\alpha)^{(1-\alpha)/2},$$

$$w = (1-\alpha)A \left(\frac{\alpha\beta(1-\gamma_0)y_0}{1-\gamma_1+\alpha\beta}\right)^{\alpha} \left(\frac{1-\alpha}{1-\gamma_1}\right)^{-\alpha/2},$$

$$R = \alpha A \left(\frac{\alpha\beta(1-\gamma_0)y_0}{1-\gamma_1+\alpha\beta}\right)^{\alpha-1} \left(\frac{1-\alpha}{1-\gamma_1}\right)^{(1-\alpha)/2}.$$

We use Example 3 to illustrate the effects of government spending. There are three types of spending shocks, current spending increases, expected future spending increases, and permanent spending increases.

Consider an increase in current spending first, modelled by an increase in γ_0. The higher spending reduces the household's disposable income in period 0 and hence reduces consumption and savings in period 0. Aggregate output and aggregate labor supply are unchanged in period 0 because factor inputs are assumed to be exogenous in period 0. In period 1, there is no shock to γ_1. Nevertheless, output falls in period 1 because the capital input is reduced due to lower savings in period 0. Consumption in period 1 is also lower for the same reason. The spending shock is again propagated from period 0 into period 1 through investment. As a result of a lower capital input and diminishing marginal product of capital, the marginal product of capital is higher and so the real interest rate is higher. Because capital and labor are complementary with each other in production, the marginal product of labor is lower. So is the real wage. Labor supply does not respond to the shock in Example 3.

Now consider an increase in future government spending, i.e. an increase in γ_1, which is anticipated at date 0. As expected, the expected increase in spending reduces the household's disposable income in period 1 and hence reduces the household's consumption in period 1. To partially counter the fall in disposable income, the household increases labor supply in period 1. These are not the only responses that the household has. Because the government spending increase is expected at date 0, the household also responds by changing consumption and savings at date 0. Anticipating that the government spending increase in period 1 will reduce the disposable income, the household reduces consumption and increases savings in period 0. That is, consumption moves in the same direction as output but investment moves in the opposite direction.

The real wage responds to the future shock ambiguously. The increase in labor supply tends to reduce the real wage, due to the diminishing marginal product of labor, but the increase in the capital input tends to increase the real wage, due to the complementarity between labor and capital in production. Similarly, the real interest rate responds to the future shock ambiguously.

We leave the analysis of a permanent government spending shock to the following exercise.

Exercise 3.4.2 *Assume* $\gamma_0 = \gamma_1 = \gamma \in (0, 1)$ *in Example 3. Analyze the effects of an increase in* γ*.*

The ambiguous response of the real wage rate to the spending shock seems a desirable feature, because the real wage in the data does not seem to be either strongly pro-cyclical or strongly counter-cyclical. However, the above effects show that government spending shocks are not the dominant driving force of business cycles. Expected government spending shocks produce two counter-factual responses. First, consumption and labor supply move in opposite directions. Second, investment is counter-cyclical. These two predictions are opposite to the observed patterns in business cycles.

The negative correlation between consumption and labor supply in period 1 is a general feature, not one specific to the utility function in Example 3. The requirement needed for such a negative correlation is that consumption and leisure are both normal, i.e., they respond to income positively. To see this, notice that the expected increase in government spending is a negative income shock. If consumption and leisure are both normal goods, they will fall in response to the negative income shock. Since labor supply is the complementary part to leisure in the household's time allocation, consumption and leisure respond in opposite directions to the government spending shock. Other fiscal policy shocks, such as changes in taxes, create similar negative correlation between consumption and labor supply.

Exercise 3.4.3 *Let the economy be described as in Example 3. Suppose that there is a permanent increase in productivity, A, and the households expect that the government increases spending in period* 1 *(through an increase in* γ_1*) to counter the productivity shock. Show that the real wage is constant if*

$$\frac{d\gamma_1}{\gamma_1} = \frac{2(1-\gamma_1)(1-\gamma_1+\alpha\beta)(1+\alpha)}{[\alpha\beta-(1-\gamma_1)]\alpha} \cdot \frac{dA}{A}.$$

What are the responses of l_1 *and* k_1 *to such a combination of changes in* A *and* γ_1*?*

Exercise 3.4.4 *Suppose that the government spending is financed by proportional income taxes, rather than lump-sum taxes. The income tax is* $\gamma_0 y_0$ *in period* 0 *and* $\gamma_1(Rk_1^s+wl_1^s)$*. Derive the equilibrium conditions and analyze the equilibrium responses to an increase in* γ_1 *that is anticipated in period* 0*.*

3.5 A Small Open Economy

The general equilibrium model is also useful for analyzing issues in open economies. In this section we leave exchange rates aside and focus on the current account. We return to the economy described in the first section of this chapter, assuming that the economy is too small relative to the world economy to have any influence on world prices of goods and capital. In this case, the world real interest rate, denoted R^*, is exogenous to the country.

In contrast to the economy described there, we require that the firms use both capital and labor to produce. Thus, savings are divided into investments in domestic capital and foreign assets. Let k_1^s be the investment in domestic capital and b the amount of foreign assets that the household owns. Then,

$$s_0 = k_1^s + b.$$

Denote the domestic real interest rate by R.

Also in contrast with the economy described in the first section of this chapter, we simplify the exercise by assuming that the labor supply decision is inelastic, each household supplying one unit of labor in each period. Setting $l = 1$ and using property (F7) of the production function, we have

$$F(k,l) = lf(k/l) = f(k). \tag{3.18}$$

Assume that the wage rate is such that the firm's net profit is zero. Labor is immobile between countries and so workers must find jobs only in the domestic country.

The household's budget constraints in the two periods are, respectively,

$$c_0 + b + k_1^s \leq y_0 - \tau_0,$$

$$c_1 \leq R^* b + R k_1^s + w - \tau_1.$$

The quantity b is the country's current account surplus in period 0 (a deficit if $b < 0$). There are two ways to see this. The first is to use the identity that the sum of the country's current account and capital account is zero. The quantity b is the country's capital account deficit in period 0 because it is the amount of capital that flows from the country into other countries. The identity then implies that b is the country's current account surplus. The second way is to interpret the current account as the country's net export. The country's output in period 0 is y_0. The part of this output that is sold within the country in period 0 is $c_0 + g_0 + k_1$. The country's net export in period 0 is $y_0 - (c_0 + g_0 + k_1)$, which is equal to b from the household's and the government's period 0 budget constraints. (The country's current account in period 1 is zero because the country does not own foreign assets in period 1.)

The household can make arbitrage between domestic capital and foreign assets. This arbitrage ensures that $R = R^*$. If $R > R^*$, the household will borrow an infinitely large amount from the world market and invest in domestic capital, which clearly is not an equilibrium. If $R < R^*$, the household will lend all to the world, which cannot be an equilibrium because the marginal product of domestic capital will be infinite in that case. With $R = R^*$, the household's intertemporal budget constraint is

$$c_0 + \frac{c_1}{R^*} \leq \left(y_0 + \frac{w}{R^*}\right) - \left(\tau_0 + \frac{\tau_1}{R^*}\right).$$

The household's and the firm's maximization problems can be formulated similarly to those in previous sections. In the equilibrium, $c_0 = y_0 - g_0 - b - k_1$ and

$$f'(k_1) = R^*, \tag{3.19}$$

$$w = f(k_1) - k_1 R^*, \tag{3.20}$$

$$\frac{U'(y_0 - g_0 - b - k_1)}{\beta U'(f(k_1) + R^* b - g_1)} = R^*. \tag{3.21}$$

Eq. (3.19) is the arbitrage-free condition $R = R^*$, where the domestic interest rate is $R = F_k = f'(k_1)$. Eq. (3.20) requires the firm's net profit to be zero. Eq. (3.21) is the condition for optimal savings, which equates the marginal rate of substitution and the marginal rate of transformation (R^*). Since labor supply is exogenously fixed at 1 in this section, there is no condition for labor supply here.

The country's current account can be determined as follows. First, we solve for the capital stock, k_1, from (3.19). Let φ denote the inverse function of f', i.e., $f'(\varphi(x)) = x$ for all x. Then (3.19) becomes $k_1 = \varphi(R^*)$. This is a decreasing function of R^*, because an increase in the world interest rate must be matched by the same amount of increase in the domestic interest rate, which is possible only if the domestic capital stock falls (due to diminishing marginal product of capital). Second, substituting $\varphi(R^*)$ into (3.21) we have

$$\frac{U'(y_0 - g_0 - b - \varphi(R^*))}{\beta U'(f(\varphi(R^*)) + R^* b - g_1)} = R^*. \tag{3.22}$$

This determines the equilibrium value of b. The left-hand side of this equation is an increasing function of b and the right-hand side does not depend on b. Thus, the solution for b is unique. We depict (3.22) in Figure 3.2, where $LHS(b)$ denotes the left-hand side of the equation.

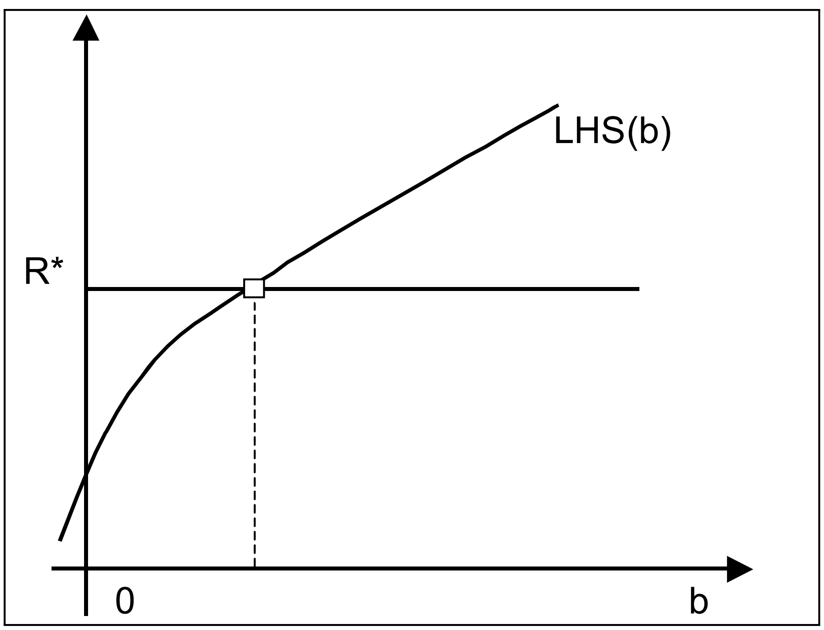

Figure 3.2. The current account in a small open economy

The condition (3.22) involves all determinants of savings analyzed in the last chapter. Thus, the factors leading to high savings tend to generate a high current account surplus. The current account is also influenced by the country's productivity and government spending. We list the effects of these factors on the current account below.

(i) The more patient toward future the country's households are, i.e., the higher β is, the higher the country's current account is. This effect is illustrated by a downward shift of the curve $LHS(b)$ in Figure 3.2. When the country's households are more patient, they save more. Since a part of savings is invested in foreign assets, the country's capital account deteriorates and the current account appreciates.

(ii) The current account is higher when the government spending profile is more tilted toward future spending. An increase in g_1 or a decrease in g_0 tilts the spending profile toward future spending. This reduces the left-hand side of (3.22), shifts the curve $LHS(b)$ downward in Figure 3.2 and increases the current account. A government profile that is tilted toward future spending reduces households' disposable income by more in the future than in the current period, thus motivating households to save more.

(iii) The current account is higher if the country has been more productive in the past or if the country will be less productive in the future. High productivity in the past increases the country's income and savings, thus improving the country's current account. Future productivity increases the marginal product of capital and attracts foreign capital to enter the country, improving the country's capital account and deteriorating the country's current account. The effect of high past productivity is represented by a high level of y_0, which shifts the curve $LHS(b)$ downward in Figure 3.2, and the effect of high future productivity is represented by a high level of $f(\varphi(R^*))$, which shifts the curve $LHS(b)$ upward. A permanent increase in productivity has ambiguous effects on the current account.

(iv) If the country does not have a large, positive current account surplus initially, an increase in the world interest rate improves the country's current account. As discussed in the last chapter, an increase in the real interest rate tends to increase savings and hence tends to improve the country's current account. This effect is represented by an upward shift of the horizontal line R^* in Figure 3.2. Also, an increase in the world interest rate induces domestic capital to flow out of the country, further improving the country's current account. This effect is represented by a downward shift of the curve $LHS(b)$ in Figure 3.2.

(v) The curve $LHS(b)$ shifts downward in (iv) only if the country does not own a large stock of foreign assets initially. If b is a sufficiently large positive number, the increase in the world interest rate increases the country's income in period 1 sufficiently through the interest payment on the international credit. As discussed in the last chapter, an increase in future income induces the household to reduce savings. If this effect is large enough to dominate the effect of the upward shift of the horizontal line R^*, the higher world interest rate can lead to a deterioration of the country's current account.

The following exercise allow us to verify the above effects in an example. It also provides a hint on how a productivity shock is transmitted into other countries.

Exercise 3.5.1 *Let* $U(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ $(\sigma > 0)$ *and* $f(k) = Ak^\alpha$ $(\alpha \in (0,1))$. *Also, let* $y_0 = Ak_0^\alpha$, $g_0 = \gamma_0 y_0$ *and* $g_1 = \gamma_1 f(k_1)$, *where* $\gamma_0, \gamma_1 \in (0,1)$. *Show that*

$$k_1 = \left(\frac{\alpha A}{R^*}\right)^{1/(1-\alpha)} \equiv \varphi(R^*),$$

$$b = \frac{(\beta R^*)^{1/\sigma}[(1-\gamma_0)Ak_0^\alpha - \varphi(R^*)] - (1-\gamma_1)f(\varphi(R^*))}{R^* + (\beta R^*)^{1/\sigma}}.$$

Use the expression for b to verify the above effects (i) – (v). In particular, find the conditions under which the current account is pro-cyclical when there is a permanent productivity shock.

Keywords in this chapter:

- Supply and demand functions of productive factors;
- General equilibrium, factor prices;
- Productivity shocks and government spending shocks (permanent and temporary), co-movement;
- Current account.