Linear Models with Rational Expectations: Problems

— Two problems —

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Problem 1

Consider that a certain economic process can be represented by the following equation

$$y_t = \alpha + \beta E_t y_{t+1} + u_t$$

where (α, β) are parameters, and u_t is an exogenous variable. $E_t y_{t+1}$ represent the expectations formulated over y_{t+1} with the available information at t.

1. Obtain a solution with rational expectations for the process y_t, assuming that $|\beta| < 1$.
2. What is the importance of parameters (α, β) for the study of the stability of this process? Explain.
3. Do you consider of any particular relevance such kind processes as described by the equation above? Explain.

Problem 1 (continued)

4. Assume now that u_t corresponds to an Autorregressive process of order 1, (AR1), given by

$$u_t = \psi + \rho u_{t-1} + \varepsilon_t \quad , \quad |\rho| < 1$$

where ψ is a constant. The term ε_t represents a series of white noise shocks (a random term with identically and independently distributed observations, having mean equal to zero and a constant variance): $\varepsilon_t \tilde{} iid(0, \delta^2)$.

5. Obtain a solution with rational expectations for the process y_t, with this additional information.

6. Obtain the same as in the previous question but now assuming that you want to obtain information about short term movements (and not about the long term equilibrium). In order to obtain this result use the following information (for simplicity assume that $\psi = 0$ in this case)

$$E_t u_{t+i} = \rho^i u_t$$

Problem 2

Consider that a certain economic process can be represented by the following system

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \lambda y_t \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) \\ i_t &= \bar{\imath} \end{aligned}$$

where (β, λ, σ) are parameters, i_t is an exogenous variable (or control variable), and $E_t(\cdot)_{t+1}$ represents expectations over $(\cdot)_{t+1}$ with the available information at t.

1. Rewrite this system in matricial form, in order to study its stability.
2. For the following set of parameters, conclude about the kind of stability we have in this system.

$$\beta = 0.8\ , \quad \lambda = 0.6\ , \quad \sigma = 2$$