System of 2 Equations: 1 Forward looking, 1 Backward looking

Forward looking:

y_t

$$\left.\begin{array}{l} E_t y_{t+1} = \alpha + \beta y_t + x_t \\ \text{or} \\ y_t = \phi_1 + \phi_2 E_t y_{t+1} + \phi_3 x_t \end{array}\right\} \text{stability requires } |\beta| > 1$$

Backward looking:

x_t

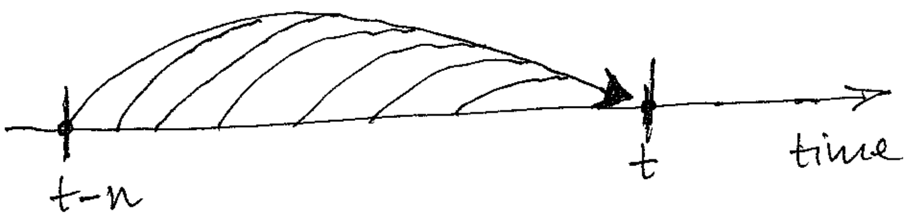
$$\underbrace{x_t = a + b\, x_{t-1} + \varepsilon_t}_{\text{stability requires } |b| < 1} \quad ; \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Backward looking vs Forward looking behavior

$$\boxed{y_{t+1} = a + b\,y_t + c\,\varepsilon_t} \quad ; \quad |b| < 1$$

$$y_t = \sum_{i=0}^{n-1} b^i a + \sum_{i=0}^{n-1} b^i c\,\varepsilon_{t-1-i}$$

$$\boxed{y_t = \frac{a}{1-b} + \sum_{i=0}^{n-1} b^i c\,\varepsilon_{t-1-i}}$$

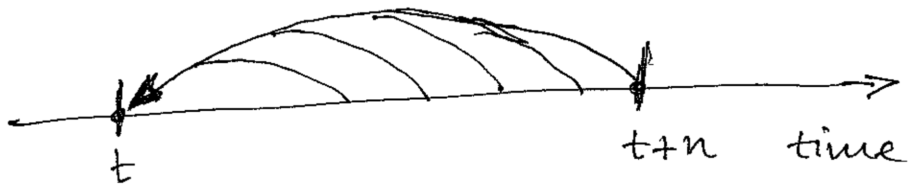


$$\boxed{E_t y_{t+1} = \alpha + \beta y_t + \varepsilon_t} \quad |\beta| > 1$$

$$y_t = a + b E_t y_{t+1} + c\,\varepsilon_t$$

with $a = -(\alpha/\beta)$, $b = (1/\beta)$, $c = -(1/\beta)$

$$\boxed{y_t = \frac{a}{1-b} + \sum_{i=0}^{n-1} b^i c\, E_t \varepsilon_{t+i}}$$



Backward looking vs Forward looking behavior

- Iteration process : if $|b|<1$ and $|\beta|>1$ we can iterate each process forward with no risk of an explosive solution

→ Knowing the dynamics of ε_t

$$\varepsilon_0 , \varepsilon_1 , \varepsilon_2 \ldots$$

→ and knowing the intial condition for each process

$$y_0$$

→ Iterating forward we will obtain (after n iterations)

$$y_1 , y_2 , y_3 \ldots y_n$$

Geometric Series

$$S = \underbrace{r^0 \cdot a + r^1 \cdot a + r^2 \cdot a + r^3 \cdot a + \dots + r^{n-1} \cdot a}_{} = \frac{a}{1-r}$$

$$= \sum_{i=0}^{n-1} r^i \cdot a$$

se $|r| < 1$ e $n \to \infty$

$$S \times r = \underbrace{r^1 \cdot a + r^2 a + r^3 \cdot a + r^4 \cdot a + \dots + r^n \cdot a}_{\text{if } |r|<1 \text{ and } n \to \infty} = \frac{a}{1-r} \cdot r$$

$$= \sum_{i=1}^{n} r^i \cdot a$$

Geometric Series

A series with a constant ratio between successive terms

$$s = r^0 \cdot a + r^1 \cdot a + r^2 \cdot a + r^3 \cdot a + \dots + r^{n-1} \cdot a$$

first term : a

common ratio : r

$$= \sum_{i=0}^{n-1} r^i \cdot a = \frac{1^{st} \text{ term}}{1 - \text{common ratio}} = \frac{a}{1-r} \quad , \text{ when } |r| < 1, \; n \to \infty$$

Now imagine that you have something that looks similar but, in fact, is slightly different :

$$s \times r = r^1 . a + r^2 . a + r^3 . a + r^4 . a + \dots + r^n . a = \sum_{i=1}^{n} r^i . a$$

Then if $s = \frac{a}{1-r}$, it is immediate to get

$$s \times r = \frac{a}{1-r} . r .$$

These are the two cases that you may encounter in our course.