Two Period Economies
Questions

Modern Macroeconomics — ISCTE–IUL — March 2012

1. Prove that the optimal intertemporal levels of consumption and savings are the same if the problem is solved in our two sequential decision making process (period by period) as explained in classes, or by using just the intertemporal consolidated budget constraint. *Hint: Instead of two constraints, we will have just one.*

2. Assume that the utility function is the following:

$$u(c_t) = \ln c_t$$

1. Show that this utility function satisfies the basic properties of a rational optimizing consumer.

2. Derive the Euler equation associated with this utility function.

3. Determine the optimal consumption levels for each period, as well as the optimal savings level.

4. Prove that the optimal consumption demand levels exhaust total intertemporal wage income. **(Do not do it)**

5. Show the conditions which have to be satisfied such that the consumer has negative savings.

3. Assume now that the utility function is of the following type:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1 - \sigma}$$

1. Demonstrate that σ is the inverse of the elasticity of substitution between current and future consumption. **(Do not do it)**

2. Answer the same questions as in the previous problem with this new utility function.

3. When $\sigma = 1$, what kind of results do we obtain?

4. Assume the same utility function as in problem 3. Consider now that the government imposes a proportional consumption tax in each period, $\tau \in (0, 1)$.

Hint: The new intertemporal consolidated budget constraint looks like:

$$(1 - \tau)c_t + \frac{(1 - \tau)c_{t+1}}{1 + r_{t+1}} = w_t + \frac{w_{t+1}}{1 + r_{t+1}}$$

5. Consider the problem of an economic agent that has to decide upon his/her intertemporal consumption. The utility function is of the following type

$$u(c_t, c_{t+1}) = c_t c_{t+1}^\alpha$$

The income constraints are the standard ones and there is some additional information: $\alpha = 0.5$, $r_{t+1} = 11\%$, and $w_t = w_{t+1} = 10$.

1. Solve analytically and graphically for the optimal levels of consumption and savings.

2. Assume now that there is the following financial constraint: $a_{t+1} = -1$. Does this constraint affect the optimal decisions of our agent? What are the new levels of optimal consumption?

3. What is the economic meaning of $a_{t+1} = 1$?

4. Write by considering the following parameter values

6. Considering the intertemporal model that we have discussed so far, discover how powerful such simple model is in explaining the behavior of some recent international macroeconomic events. Assume that a country is populated by a large number of a representative household as the one that we have been discussing in our classes.

1. What is expected to happen to a country that experiences a strong and persistent decline in the cost of borrowing?

2. What is expected to happen to the savings in a country that experiences a strong and persistent decline in the cost of borrowing?

3. What is expected to happen to a country that experiences the breakdown of strong financial constraints?